RECIPROCAL FREQUENCY NOISE EXPLAINED BY HIGHER SYMMETRY ELECTRODYNAMICS

ABSTRACT

A higher symmetry form of electrodynamics provides a straightforward explanation of reciprocal frequency (1/*f*) noise.

The inference and empirical verification of quarks in contemporary gauge field theory {1-5} depends on a non-Abelian symmetry group (SU(3)). In this paper, these powerful ideas are applied to electrodynamics to produce a straightforward explanation of an important phenomenon of the solid-state electronics, the reciprocal frequency noise (1/*f* noise). The 1/*f* spectrum occurs primarily in quantum electronic systems and it is thought that the underlying process is non-linear {6}. It has been discovered recently in systems completely unrelated to quantum electronics {7} and is perhaps indicative of a general property of nature.

The explanation of the phenomenon offered here depends on the simplest type of non-Abelian Hamiltonian for the interaction of electron and photon. Its semi-classical {8, 9} equivalent is:

$$H = -\frac{e\hbar}{2m}\left(\mathrm{A}\bullet\mathrm{A}^{*} + i\sigma\bullet\mathrm{A}\times\mathrm{A}^{*}\right) \tag{1}$$

where e/m is the charge to mass ratio of the electron, $\hbar$ is the Dirac constant, σ is the Pauli matrix and A is the complex valued electromagnetic field potential. The second term in eqn. (1) gives rise to fermion resonance induced by circularly polarized radiation. In quantum electrodynamics, this Hamiltonian can be used for the description of the 1/*f* noise problem through creation and annihilation operators describing the exchange of a phonon: the absorption of a phonon in state p and its emission in state $p + k'$. The electron changes state from k to $k + k'$. The specifically non-Abelian term $\left(-\frac{e\hbar}{2m}i\sigma\bullet\mathrm{A}\times\mathrm{A}^{*}\right)$ in the semi-classical Hamiltonian (1) describes a novel non-Abelian interaction between phonons and electrons in a lattice. The frequency dependence in this interaction is $\left(\omega_{k}\omega_{k-q}\right)^{1/2}$, and can be approximated by **the reciprocal frequency** $(\omega_{k'})^{-1}$ if $\omega_{0_v} \ll \omega_k$ and if we sum over $k \rightarrow k' + \frac{q}{2}$ in this approximation.

The appearance of reciprocal noise is therefore a straightforward consequence of the non-Abelian term in the Hamiltonian (1), a term which may be represented as the magnetic interaction {8, 9}:

$$H = -\frac{e\hbar}{2m}\sigma\bullet\boldsymbol{B}^{(3)}. \tag{2}$$

In our approximation, the electrons and phonons interact weakly - only a small fraction of phonon momentum is transferred to the electron. When the momentum k is fixed, the picture being drawn is that of a set of phonons interacting with the electrons with one "input" set of electrons in one mode. It follows {10} that the expectation energy of the electrons has the 1/*f* curve characteristic of noise encountered in quantum electronics.

This straightforward explanation of $1/f$ noise depends on the non-Abelian nature of the Hamiltonian, whose semi-classical form contains the interaction between the magnetic field $B^{(3)}$ and the spin angular momentum of the electron. The $B^{(3)}$ field {1-5, 8-10} is the archetypical spin field in the higher symmetry form of electrodynamics in which the field equations are those of an O(3) symmetry gauge field theory. This higher symmetry electrodynamics is already known to have several major advantages over the Maxwell-Heaviside theory. It is intrinsically non-linear and so provides a natural and simple explanation of the $1/f$ noise from the first principles of gauge theory {11, 12}. This is a major advance in our understanding of this process in quantum electronic systems and also in our understanding of quantum electrodynamics.

In standard theory {6}, $1/f$ noise is caused by the interaction between electrons and lattice defects and is modeled according to electron phonon interactions by assuming that lattice defects induce bunching or aggregation of lattice atoms. It is thought that this also causes phonons to aggregate and that this process is equivalent to the bunching of photons in a non-linear medium. The relaxation rate of the lattice is analogous to the dispersion of photons, a process analogous in turn to non-linear atomic response to photons.

This approach contains the germ of the explanation offered in this paper, because the occurrence of lattice defects is modeled intrinsically by considering the phonon to be a non-linear object with non-Abelian properties. In this paper, these non-linear phonons have been considered to be basic to the physics, and that the electrons interact with a bath of non-linear phonons according to the principles of non-Abelian gauge theory. The phonons exist in a thermal distribution, and so it follows that the electrons exist in a distribution of states that reflect Fermi-Dirac statistics, leading directly to the $1/f$ noise spectrum.

The adoption of a non-Abelian gauge theory to explain $1/f$ noise is based on a similar conceptual advance to a higher symmetry electrodynamics wherein the usual Maxwell-Heaviside theory becomes a gauge theory with non-Abelian background symmetry. The higher symmetry electrodynamics is characterized by a non-vanishing commutator of potentials which gives rise to the characteristic magnetic field, $B^{(3)}$ appearing in eqn. (1). From this conceptual advance, it is possible to derive the non-linear wave equations such as the non-linear Schrödinger equation that describe photon bunching and vortex effects.

The complex interactions between optical phonons that occur in a solid with lattice disorders have been modeled in this paper according to this higher symmetry electrodynamics. The phonons originate in lattice vibrations associated with charge separation of anions and cations, and the essence of the method employed in this paper is to apply non-Abelian algebra to the electric and magnetic fields associated with these phonons. The electronic properties of a solid are determined by the interaction of phonons with electrons, the interaction of lattice dynamics and electronic quantum states. When the interaction between phonon and electron is linear, the absorption of a phonon with a certain linear momentum causes an electronic state transition. This process is precisely analogous to the absorption of a photon by an atom. The Fermi-Dirac statistics of the electrons interact with the Bose-Einstein statistics of the phonons.

When the theory is upgraded so that the phonons take on a non-Abelian structure, an interaction of the type (1) appears between the $B^{(3)}$ field and the electronic angular momentum, an interaction given by the Sakurai equation. On the level of quantum electrodynamics, this interaction is quadratic in the electron and phonon raising and lowering operators. The interaction Hamiltonian becomes a sum over fermion and phonon modes that corresponds to the absorption of a phonon by the electron and the emission of another phonon. This process changes the momenta of phonon and electron and leads to the $1/f$ noise spectrum as follows.

The above process occurs for a statistical ensemble of electrons. The electrons in the conduction band of a solid will occupy states one at a time according to Fermi-Dirac statistics. In the solid, there is a gas of electrons that obey these statistics where, on average, for every electron that absorbs a certain linear momentum from the phonons, there will be another that will lose the same amount of linear momentum. The electrons and phonons are in equilibrium in detailed balance where each state is given by a Boltzmann factor.

A sum over these fermions leads to the expectation energy for the occurrence of a phonon, and a reciprocal relation between the frequency of an electron and the expectation value of its energy - the $1/f$ spectrum.

The essence of this paper therefore is the straightforward solution of this intractable problem. The complex interaction between phonon and electron in a lattice with defects is modeled in analogy with the description of non-linear optical properties from the interaction of electron and a higher symmetry quantized electromagnetic field.

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