

## FERMION RESONANCE INDUCED BY CIRCULARLY POLARIZED MICROWAVE RADIATION WITHOUT PERMANENT MAGNETIC FIELDS

### ABSTRACT

It is shown, using several methods, that circularly polarized microwave radiation induces fermion resonance which is proportional to the power density divided by the square of the angular frequency of the microwave radiation. By adjusting conditions, the resonance can be made to occur at much higher frequencies than those obtainable with contemporary superconducting magnet technology, and occurs without the use of permanent magnets due to the interaction of the conjugate product of the field with the Pauli matrix representing the half integral spin of the fermion. The occurrence of the fermion resonance at much higher frequencies than those achievable with permanent magnets means that the resolution of the spectrum is increased commensurably. In principle therefore, a high resolution fermion resonance spectrum is obtainable with its own spectral fingerprint.

### INTRODUCTION

The resolution of current nuclear magnetic resonance (NMR) and electron spin resonance (ESR) instruments is limited by the magnetic flux density produced by the permanent magnets which are used to produce the resonance spectrum {1-3}. This instrumental limitation is present in all types of NMR and ESR and limits the spectral resolution. In this paper, it is inferred theoretically that circularly polarized microwave radiation produces electron or proton resonance at a much higher frequency than that obtainable with even the most powerful superconducting magnet now available, so the spectral resolution is increased correspondingly. This effect can be achieved without the use of a permanent magnet and the chemical shift pattern is unique, because it depends on a novel property tensor. Population differences between up and down states of the fermion spin are large enough that inhomogeneity in the pump beam is not a restriction, and the spectrum is site specific. The apparatus is simple to design and manufacture in principle and produces a resonance spectrum, in theory, with a microwave beam interacting with a fermion beam. The theory of the effect is developed in this paper from first principles using equations from the classical to quantum non-relativistic levels. In each case, the result is obtained that fermion resonance can be induced directly by a circularly polarized microwave beam and that this resonance is proportional to power density divided by the square of angular frequency.

In Section 2, the minimal prescription is used with a complex vector potential to demonstrate the effect at the classical level for the unshielded electron and proton. In Section 3, the non-relativistic Schrödinger equation with intrinsic spin is used to demonstrate its presence in quantum mechanics. In Section 4, relativistic equations are used to confirm the results of Sections 2 and 3. Finally, a discussion is given of the technological aspects of the experiment, and an optimum design suggested.

### CLASSICAL DERIVATION OF THE EFFECT

The classical derivation of the effect is a simple one. The kinetic energy of a classical electron is rewritten in the SU(2) basis as:

$$T = \frac{1}{2m} \mathbf{p} \cdot \mathbf{p} = \frac{1}{2m} (\boldsymbol{\sigma} \cdot \mathbf{p})(\boldsymbol{\sigma} \cdot \mathbf{p}) \quad (1)$$

where  $e/m$  is the charge to mass ratio of the classical electron, and where  $p$  is its linear momentum. There is no objection to this procedure on the classical level because the covering group of SU(2) is O(3), the rotation group symmetry. The interaction of the classical electron and the classical electromagnetic field is described by:

$$T = \frac{1}{2m} \sigma \cdot (p - eA) \sigma \cdot (p - eA^*) \quad (2)$$

where  $A$  is the complex valued vector potential. The term responsible for radiatively induced fermion resonance (RFR) emerges from this equation straightforwardly. Averaging over many cycles of the field produces the result:

$$\langle T \rangle = \frac{1}{2m} p \cdot p + \frac{e^2}{2m} \sigma \cdot A \sigma \cdot A^* \quad (3)$$

and the mean interaction energy is:

$$\langle T \rangle_{\text{int}} = \frac{e^2}{2m} \sigma \cdot A \sigma \cdot A^* \quad (4)$$

This expression can be expanded as:

$$\langle T \rangle_{\text{int}} = \frac{e^2}{2m} (A \cdot A^* + i \sigma \cdot A \times A^*) \quad (5)$$

and the RFR term is the second on the right hand side:

$$T_{RFR} = i \frac{e^2}{2m} \sigma \cdot A \times A^* \quad (6)$$

The existence of electron spin resonance is deduced from the usual resonance condition:

$$T_{RFR} = \hbar \omega_{\text{res}} \quad (7)$$

where  $\hbar \omega_{\text{res}}$  is the energy of one photon of a probe beam {4}. By using the equation for electromagnetic intensity or power density  $I$  (watts/cm<sup>2</sup>):

$$I = \frac{c}{\mu_0} B^{(0)2} \quad (8)$$

where  $\mu_0$  is the vacuum permeability, it is easily deduced that electron resonance occurs at a frequency:

$$f_{\text{res}} = \frac{\omega_{\text{res}}}{2\pi} = \left( \frac{e^2 \mu_0 c}{2\pi \hbar m} \right) \frac{I}{\omega^2} = 1.007 \times 10^{28} \frac{I}{\omega^2} \quad (9)$$

By adjusting  $I/\omega^2$ , the resonance frequency can be tuned to the infrared or visible range compared with the MHz range of magnet based ESR {5}. This advantage brings with it a concomitant increase in resolution of several orders of magnitude over conventional ESR, with great technical advantages in principle. The challenge is technological rather than theoretical. An estimate of the radiatively induced NMR spectrum can be made by allowing for the different nuclear  $g$  factor of proton and electron {6}, resulting in:

$$\omega_{res} = \left( \frac{5.5857 e^2 \mu_0 c}{2.002 \hbar m} \right) \frac{I}{\omega^2} = 1.532 \times 10^{25} \frac{I}{\omega^2} \quad (10)$$

## QUANTUM MECHANICAL DERIVATION OF THE EFFECT

To describe the intrinsic spin of the electron or proton in non-relativistic quantum mechanics, the method described by Sakurai {7} is used. The Hamiltonian in the absence of the electromagnetic field is

$$H = \frac{1}{2m} (\boldsymbol{\sigma} \cdot \mathbf{p})(\boldsymbol{\sigma} \cdot \mathbf{p}) + V \quad (11)$$

and the equation of motion is the non-relativistic Schrödinger Pauli equation:

$$H\psi = E\psi \quad (12)$$

In the presence of a static magnetic field, the vector potential is real valued and the Hamiltonian becomes:

$$H = \frac{1}{2m} (\boldsymbol{\sigma} \cdot (\mathbf{p} + e\mathbf{A})) (\boldsymbol{\sigma} \cdot (\mathbf{p} + e\mathbf{A})). \quad (13)$$

This results in the famous “half integral spin” term:

$$H\psi = \left( \frac{e\hbar}{2m} \boldsymbol{\sigma} \cdot \mathbf{B} \right) \psi \quad (14)$$

responsible for all ESR, NMR and MRI with permanent magnetic fields.

With complex valued  $\mathbf{A}$  (electromagnetic field) in eqn. (13), we obtain the purely classical RFR term:

$$H(RFR) = i \frac{e^2}{2m} \boldsymbol{\sigma} \cdot \mathbf{A} \times \mathbf{A}^* \quad (15)$$

and confirm that the non-relativistic classical and non-relativistic quantum mechanical derivations give the same result.

The resonance condition in RFR occurs under the condition

$$\hbar\omega_{res} = \frac{e^2 c^2 B^{(0)2}}{2m\omega^2} (1 - (-1)) \quad (16)$$

where  $\omega$  is the angular frequency of a probe electromagnetic field. For an electron, the resonance frequency in hertz is therefore given by {8}:

$$\omega_{res(\text{electron})} = 1.007 \times 10^{28} \frac{I}{\omega^2} \quad (17)$$

and for a proton, we adjust empirically for the fact that the  $g$  factor of the proton is 5.5857, compared with 2.002 for the electron. The proton resonance frequency in RFR is therefore {8}:

$$\omega_{res(\text{proton})} = 1.532 \times 10^{25} \frac{I}{\omega^2}. \quad (18)$$

## EXPERIMENTAL TESTS OF THE THEORY.

The theory can be tested under ideal conditions with the use of an electron or proton beam, and the dependence of the resonance frequency on  $I/\omega^2$  should be detectable empirically. In the absence of such data, there are a few experiments in the literature which can be used to test eqn. (18). Pershan et al., {9} in their empirical demonstration of the inverse Faraday effect, show the presence of a term proportional to  $i\sigma \cdot A \times A^*$  in perturbation theory, a term which was observed experimentally in the rare earth doped glasses at low temperature. Warren et al. {10} report sub-Hertzian shifts in molecular liquids subjected to irradiation with a circularly polarized argon ion laser at 528.7 nm, 488 nm and 476.6 nm with a power density of 10 watts/cm<sup>2</sup>. Under these conditions, the resonance frequencies from eqn. (18) are 0.12, 0.10 and 0.098 Hz. However the data by Warren et alia {10} were obtained at extreme instrumental limits and they did not find the expected  $I/\omega^2$  dependence or expected alignment of spins by a circularly polarized laser. However, the latter phenomenon is well known in the phenomenon of optical NMR {11} using a Fermi contact mechanism, and this suggests that if the RFR experiment were carried out under the optimum conditions (microwave beam, electron beam), the expected  $I/\omega^2$  dependence would emerge.

The design of the two beam experiment is critical, because the microwave radiation has to be accurately circularly polarized with circular waveguides which must guide the microwave radiation into the electron beam under optimum conditions, which probably must be found by trial and error. The paper by Deschamps et al., {12} in which the inverse Faraday effect was observed with circularly polarized microwave radiation, contains details of design. Without such careful design considerations, it is unlikely that the experiment will be successful. If the experiment on the electron beam is successful, work can be extended to a proton and atomic beam.

## THE CHEMICAL SHIFT IN RFR

In addition to ultra high resolution, the technique of RFR is site selective in a unique way, because the chemical shift is determined by a different molecular property tensor from that effective in NMR and ESR. The chemical shift in RFR can be calculated using the second order perturbation energy (S.I. units):

$$E_n = \sum \frac{\langle 0|H|n\rangle\langle n|H|0\rangle}{\hbar\omega_{0n}} \quad (19)$$

with the perturbation Hamiltonian:

$$H = \frac{1}{2m}(\mathbf{p} + e(\mathbf{A} + \mathbf{A}_N))^2 + V \quad (20)$$

where

$$\mathbf{A}_N = \frac{\mu_0}{4\pi r^3} \mathbf{m}_N \times \mathbf{r} \quad (21)$$

is the vector potential {13} due to the nuclear magnetic dipole moment  $\mathbf{m}_N$ . The perturbation term relevant to the RFR chemical shift is the dominant single photon off resonance population term:

$$E_n = \frac{ie^3}{m^2 \hbar\omega_{0n}} \sum \langle 0|\mathbf{p} \cdot \mathbf{A}|n\rangle\langle n|\mathbf{A}_N \cdot \mathbf{A}^*|0\rangle + \text{c.c.} \quad (22)$$

The electronic electric transition dipole moment is defined by {14}:

$$\langle 0|\boldsymbol{\mu}|n\rangle = \frac{e}{m\omega_{0n}} \langle 0|\mathbf{p}|n\rangle. \quad (23)$$

Using the vector relation:

$$\boldsymbol{\mu} \times (\mathbf{m}_N \times \mathbf{r}) = (\boldsymbol{\mu} \cdot \mathbf{r}) \mathbf{m}_N - (\boldsymbol{\mu} \cdot \mathbf{m}_N) \mathbf{r} \quad (24)$$

we find that the term (22) can be written as

$$En = \zeta \left( i \frac{e^2}{2m} \boldsymbol{\sigma} \cdot \mathbf{A} \times \mathbf{A}^* \right) \quad (25)$$

where

$$\zeta = \frac{g_N e \mu_0}{8\pi m} \sum \langle 0 | \boldsymbol{\mu} | n \rangle \langle n | \frac{\mathbf{r}}{r^3} | 0 \rangle \quad (26)$$

with

$$\mathbf{m}_N = g_N \frac{e}{4m} \hbar \boldsymbol{\sigma}. \quad (27)$$

The factor  $\zeta$  is the RFR chemical shift or shielding constant. It depends on the novel molecular property tensor in eqn. (26), novel in the sense that it is not the same molecular property tensor that defines the chemical shift in NMR {14} through the well known Lamb formula. Its order of magnitude in the first approximation is however  $10^{-6}$ , roughly the same as in NMR. The complete RFR spectrum from the protons in atoms and molecules is therefore, from eqns. (25) and (15):

$$T_{RFR} = i \frac{e^2}{2m} (1 + \zeta) \boldsymbol{\sigma} \cdot \mathbf{A} \times \mathbf{A}^* \quad (28)$$

and is site specific because  $\zeta$  is site specific.

## DISCUSSION

The existence of RFR has been shown straightforwardly on the classical and quantum levels in the non-relativistic limit. It can be shown that the effect also exists in the appropriate relativistic equations {15} such as the Dirac equation. In theory, a circularly polarized microwave beam can produce electron and proton resonance at much higher frequencies than possible from even the most powerful superconducting magnet, and RFR has its unique spectral fingerprint, giving it a double advantage. It is the first purely radiative method of inducing fermion resonance, and does not need a permanent magnet. It is easy to calculate the expected resonance frequencies from eqns. (17) and (18), which are equations for the bare electron and bare proton. In atomic or molecular beams, a more complicated spectrum is expected from eqn. (28), i.e. there is expected to be a chemical shielding effect. Finally, the theory of RFR can be developed for atoms and molecules with more than one nucleus, because the resonance frequency is expected to be different for each nucleus.

## ACKNOWLEDGMENTS

Many colleagues worldwide are thanked for internet discussions and the constituent institutions of AIAS (Alpha Foundation's Institute for Advanced Study) for funding contributions.

## REFERENCES

- {1} R.R. Ernst, G. Bodenhausen and A. Wokaun, "Principles of Nuclear Magnetic Resonance in One and two Dimensions" (Oxford University Press, Oxford, 1987).
- {2} P.A. M. Dirac, "Quantum Mechanics" (Oxford Univ. Press, Oxford, 1974, 4<sup>th</sup>. Ed.).

- {3} M.W. Evans, J.P. Vigier, S. Roy and S. Jeffers, "The Enigmatic Photon, Volume Three, Theory and Practice of the  $B^{(3)}$  Field." (Kluwer Academic, Dordrecht, 1996), chapters one and two.
- {4} P.W. Atkins, "Molecular Quantum Mechanics" (Oxford Univ. Press, Oxford, 1983, 2<sup>nd</sup>. Ed.).
- {5} D. Gammon, S. W. Brown, E. S. Snow, T. A. Kennedy, D. S. Katzer and D. Park, *Science*, **277**, 85 (1997).
- {6} A list of nuclear  $g$  factors is given in ref. (4).
- {7} J.J. Sakurai, "Advanced Quantum Mechanics" (Addison Wesley, New York, 1967, 11<sup>th</sup> printing), chapter 3.
- {8} M.W. Evans, "The Enigmatic Photon, Volume Five,  $O(3)$  Electrodynamics" (Kluwer Academic, Dordrecht, 1999), chapter 3.
- {9} P.S. Pershan, J.P. van der Ziel and L.D. Malmstrom, *Phys. Rev.*, **143**, 574 (1966).
- {10} W.S. Warren, S. Mayr, D. Goswami, and A.P. West, Jr., *Science*, **255**, 1683 (1992); **259**, 836 (1993); *Mol. Phys.*, **93**, 371 (1998); M.W. Evans, *Found. Phys. Lett.*, in press, 1999.
- {11} S.W. Brown, T.A. Kennedy and D. Gammon, *Solid State N.M.R.*, **11**, 49 (1998).
- {12} J. Deschamps, M. Fitaire and M. Lagoutte, *Phys. Rev. Lett.*, **25**, 1330 (1970); *Rev. Appl. Phys.*, **7**, 155 (1972).
- {13} R.A. Harris and I. Tinoco, *J. Chem. Phys.*, **101**, 9289 (1994).
- {14} Atkins in ref. (4) contains a clear account of the chemical shift in NMR.
- {15} The relativistic derivation of RFR is given in refs (3) and (8).