NOTES

THE MEANING OF BARRETT'S NOTATION

Reference: p. 299, Table 2, of Barrett and Grimes, "Advanced Electromagnetism" (World Scientific, 1995)

The SU(2) Coulomb Law (Gaussian Units)

$$\nabla \bullet \boldsymbol{E} = J_0 = iq\left(\boldsymbol{A} \bullet \boldsymbol{E} - \boldsymbol{E} \bullet \boldsymbol{A}\right) \tag{1}$$

Now Rodrigues claims that Barrett equates a scalar J_0 with a tensor. This is incorrect. The Barrett notation is in the SU(2) basis, using Pauli matrices. So we can break out eqn. (1) as follows. First, note from Barrett's table (1), p. 298, that:

$$\boldsymbol{J}_Z = \boldsymbol{A} \bullet \boldsymbol{E} - \boldsymbol{E} \bullet \boldsymbol{A} \tag{2}$$

This means:

$$\begin{bmatrix} J_Z & 0 \\ 0 & -J_Z \end{bmatrix} = \begin{bmatrix} A_Z & A_X - iA_Y \\ A_X + iA_Y & -A_Z \end{bmatrix} \begin{bmatrix} E_Z & E_X - iE_Y \\ E_X + iE_Y & -E_Z \end{bmatrix} - \begin{bmatrix} E_Z & E_X - iE_Y \\ E_X + iE_Y & -E_Z \end{bmatrix} \begin{bmatrix} A_Z & A_X - iA_Y \\ A_X + iA_Y & -A_Z \end{bmatrix}$$

i.e.

$$J_Z = 2i\left(E_Y A_X - E_X A_Y\right) \tag{3}$$

$$E_Z A_Y - A_Z E_Y = 0 \tag{4}$$

The **Lehnert Charge** is:

$$\rho_L = -iqJ_Z = 2q\left(E_Y A_X - E_X A_Y\right)$$

So eqn. (1) becomes:

$$\nabla \bullet \mathrm{E} = J_0 + 2q\left(E_Y A_X - E_X A_Y\right) \tag{5}$$

in **Gaussian** units. Specifically:

$$\frac{\partial E_Z}{\partial Z} = J_0 + 2q\left(E_Y A_X - E_X A_Y\right) \tag{6}$$

This is obviously a special case of the more general O(3) Coulomb Law (in S.I. units):

$$\nabla \bullet \mathrm{E}^{(1)*} = \frac{\rho^{(1)*}}{\varepsilon_0} + ig\left(A^{(2)} \bullet \mathrm{E}^{(3)} - \mathrm{E}^{(2)} \bullet A^{(3)}\right) \tag{7}$$

et cyclicum.

The **Lehnert charges** being:

$$\rho_L^{(1)*} = ig\left(A^{(2)} \bullet E^{(3)} - E^{(2)} \bullet A^{(3)}\right) \tag{8}$$

et cyclicum.

We note the complete structural agreement between eqn (6) (Barrett), and eqns. (7), **and** with Lehnert's hypothesis.