FPL submitted
At its website

# DERIVATION OF A LOCALLY GAUGE INVARIANT PROCA EQUATION FROM U(1) AND O(3) GAUGE THEORY APPLIED TO VACUUM ELECTRODYNAMICS: ACQUISITION OF PHOTON MASS AND REST ENERGY FROM THE VACUUM.

by

Petar K. Anastasovski (1), T. E. Bearden (2), C. Ciubotariu (3), W. T. Coffey (4), L. B. Crowell (5), G. J. Evans (6), M. W. Evans (7,8), R. Flower (9), A. Labounsky (10), B. Lehnert (11), M. Mészáros (12), P. R. Molnár (12), S. Roy (13), and J.-P. Vigier (14).

Institute for Advanced Study, Alpha Foundation, Institute of Physics, 11 Rutafa Street, Building H, Budapest, H-1165, Hungary

Also at:

1) Faculty of Technology and Metallurgy, Department of Physics, University of Skopje, Republic of Macedonia;

2) CEO, CTEC Inc, 2311 Big Cove Road., Huntsville, AL 35801-1351.

PHYSICA SCRIPTA
Manus No. 9116
Received. 00.04.09
Accepted. ..........

3) Institute for Information Technology, Stuttgart University, Stuttgart, Germany;

4) Department of Microelectronics and Electrical Engineering, Trinity College, Dublin 2, Ireland;

5) Department of Physics and Astronomy, University of New Mexico, Albuquerque, New Mexico;

6) Ceredigion County Council, Aberaeron, Wales, Great Britain.

7) former Edward Davies Chemical Laboratories, University College of Wales, Aberystwyth SY32 1NE, Wales, Great Britain;

8) sometime JRF, Wolfson College, Oxford, Great Britain;

9) CEO, Applied Science Associates and Temple University, Philadelphia, Pennsylvania, USA.

10) The Boeing Company, Huntington Beach, California,

11) Alfven Laboratory, Royal Institute of Technology, Stockholm, S-100 44, Sweden,

12) Alpha Foundation, Institute if Physics, 11 Rutafa Street, Building H, Budapest, H-1165, Hungary.

13) Indian Statistical Institute, Calcutta, India;

14) Labo de Gravitation et Cosmologie Relativistes, Université Pierre et Marie Curie, Tour 22-12, 4 ème étage, BP 142, 4 Place Jussieu, 75252 Paris Cedex 05, France

---

ABSTRACT

Electrodynamics in the vacuum is considered as a U(1) and O(3) invariant gauge theory. In both cases local gauge transformation results in a vacuum charge / current density. A Higgs mechanism is used to derive a locally gauge invariant Proca equation in a U(1) and O(3) invariant electrodynamics. Therefore the photon acquired mass and rest energy from the vacuum as it propagates. The advantages of an O(3) over a U(1) invariant gauge theory applied to vacuum electrodynamics are discussed.

KEYWORDS: Locally gauge invariant Proca equation; vacuum charge / current density; photon mass; Higgs mechanism.

## 1. INTRODUCTION

Gauge theory applied to electrodynamics in the vacuum requires the presence of an internal gauge space {1-3}. In a U(1) invariant gauge theory of any type the internal space is characterized by a complex field with two components in a plane and in an O(3) invariant theory by a complex field with three components in three dimensional space. The use of a complex field indicates that the particle concomitant with the field is charged. These concepts may be applied to electromagnetism in the vacuum by considering a topological charge, g, which appears in the covariant derivative obtained by local gauge transformation and by considering the components of the field in the internal space to be components of the vector potential in the vacuum. From this starting point a globally invariant lagrangian is constructed in Section 2 both on the U(1) and O(3) invariant levels. From this lagrangian, the wave equation in the internal space of the gauge theory is obtained in the vacuum. ~~In section 3~~ The locally invariant lagrangian is obtained from a local gauge transformation and Euler-Lagrange equations used to derive the locally gauge invariant wave and field equations. The latter contain a vacuum charge current density both in a U(1) and O(3) invariant gauge theory. This type of vacuum charge current density was first inferred by Lehnert {4 - 6}. In Section ~~4~~ 3 a Higgs mechanism is used to derive a locally gauge invariant Proca equation and a photon mass defined by spontaneous symmetry breaking of the vacuum in a U(1) and O(3) invariant gauge theory. Finally the advantages of the O(3) over the U(1) invariant gauge theories applied to electrodynamics are discussed in detail.

## 2. GLOBALLY INVARIANT LAGRANGIANS AND WAVE EQUATIONS IN THE INTERNAL SPACES OF THE U(1) AND O(3) INVARIANT GAUGE THEORIES.

The theory is first developed in U(1) invariant form {1 - 3} by considering a complex field made up of two components of the vector potential:

$$A = \frac{1}{\sqrt{2}}\left(A_1 + iA_2\right) \qquad — (1)$$

$$A^* = \frac{1}{\sqrt{2}}\left(A_1 - iA_2\right) \qquad — (2)$$

The complex field A and its conjugate $A^*$ are considered to be independent fields, and signal the existence of a topological charge {7 - 12}:

$$g = \frac{\kappa}{A^{(0)}} \qquad — (3)$$

where $\kappa$ is the wave-number and $A^{(0)}$ a vector potential magnitude. The topological charge defines the U(1) invariant covariant derivative:

$$D_\mu = \partial_\mu + igA_\mu \qquad - (3)$$

after local gauge transformation {1 – 3}. The complex fields ( 1 ) and ( 2 ) define the globally invariant lagrangian:

$$\mathcal{L} = (\partial_\mu A)(\partial^\mu A^*) \qquad - (4)$$

and the Euler Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial A} = \partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu A)} \right); \quad \frac{\partial \mathcal{L}}{\partial A^*} = \partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu A^*)} \right) - (5)$$

produce d'Alembert wave equations for each component:

$$\Box A = \Box A^* = 0. \qquad - (6)$$

Local gauge transformation is now applied, defined by:

$$A \rightarrow \exp(-i\Lambda(x^\mu))A; \quad A^* \rightarrow \exp(i\Lambda(x^\mu))A^* \qquad - (7)$$

to give the locally gauge invariant lagrangian {1 – 3}

$$\mathcal{L} = D_\mu A D^\mu A^* - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \qquad - (8)$$

from which the Euler Lagrange equations ( 5 ) ~~and ( )~~ give the

locally gauge invariant wave equations in the vacuum:

$$D^\mu \left( D_\mu A \right) = 0 \qquad - (9)$$

$$D_\mu \left( D^\mu A^* \right) = 0 \qquad - (10)$$

where $A_\mu$ is the four potential and where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad - (11)$$

is the electromagnetic field in the vacuum.

The Euler Lagrange equation:

$$\partial_\nu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\nu A_\mu)} \right) = \frac{\partial \mathcal{L}}{\partial A_\mu} \qquad - (12)$$

gives the Lehnert equation {4–6} in S.I. units:

$$\partial_\nu F^{\mu\nu} = -igc \left( A^* D^\mu A - A D^\mu A^* \right) \qquad - (13)$$

where $D_\mu$ is the covariant derivative defined by { 1–3 }:

$$D_\mu A = \left( \partial_\mu + igA_\mu \right) A \qquad - (14)$$

$$D_\mu A^* = \left( \partial_\mu - igA_\mu \right) A^* \qquad - (15)$$

Therefore if U(1) invariant gauge theory is applied rigorously to electromagnetism in the vacuum there appears a vacuum charge current density:

$$J^{\mu}(vac) = -i\epsilon_0 gc\left(A^* D^{\mu} A - A D^{\mu} A^*\right) \quad -(16)$$

and the wave equations become ( 9 ) and ( 10 ). A covariant derivative appears containing the topological charge ( 3 ).

The basis of this development is that in a U(1) invariant gauge there must be an internal space of this symmetry. The only field present is the electromagnetic field, so the internal space must define the complex scalar fields ( 1 ) and ( 2 ).

In the usual Maxwell Heaviside theory, the Lehnert charge current density ( 16 ) is missing, and there is no topological charge present in the theory, so covariant derivatives are replaced by ordinary derivatives and the vacuum d'Alembert equations ( 9 ) and ( 10 ) become:

$$\Box A^{\mu} = 0 \quad -(17)$$

$$\Box A = 0 \quad -(18)$$

$$\Box A^* = 0 \quad -(19)$$

There is no internal gauge space present in the Maxwell Heaviside theory and therefore the theory cannot be described self-consistently as U(1) invariant in the vacuum. The structure of the wave equations ( 9 ) and ( 10 ) do not correspond to that of a Proca equation.

In an O(3) invariant vacuum electrodynamics {7 - 12} the globally invariant lagrangian is:

$$\mathcal{L} = \partial_\mu \underline{A} \cdot \partial^\mu \underline{A}^* \qquad - (20)$$

where $\underline{A}$ and $\underline{A}^*$ are vectors in the O(3) symmetry internal gauge space of the theory. These are independent complex vectors and the concomitant charge is again the topological charge defined in eqn. ( 3 ). The Euler Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial \underline{A}} = \partial_\nu \left( \frac{\partial \mathcal{L}}{\partial_\nu \underline{A}} \right) ; \quad \frac{\partial \mathcal{L}}{\partial \underline{A}^*} = \partial_\nu \left( \frac{\partial \mathcal{L}}{\partial_\nu \underline{A}^*} \right) \qquad - (21)$$

give the globally invariant d'Alembert wave equations:

$$\Box \underline{A} = \underline{0} ; \quad \Box \underline{A}^* = \underline{0} . \qquad - (22)$$

A local gauge transformation on the O(3) level is defined by {7 - 12}:

$$\underline{A} \rightarrow e^{iJ_i\Lambda_i}\underline{A} \; ; \; \underline{A}^* \rightarrow e^{-iJ_i\Lambda_i}\underline{A}^* \quad -(23)$$

where $J_i$ are rotation generators of the O(3) group and where $\Lambda_i$ are angles. The gauge transformation ( 23 ) gives the locally gauge invariant lagrangian:

$$\mathcal{L} = D_\mu \underline{A} \cdot D^\mu \underline{A}^* - \frac{1}{4} \underline{G}_{\mu\nu} \cdot \underline{G}^{\mu\nu} \quad -(24)$$

The Euler Lagrange equation:

$$\frac{\partial \mathcal{L}}{\partial \underline{A}_\mu} = \partial_\nu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\nu \underline{A}_\mu)} \right) \quad -(25)$$

gives the O(3) invariant inhomogeneous field equation in the vacuum:

$$D_\nu \underline{G}^{\mu\nu} = -g D^\mu \underline{A}^* \times \underline{A} \quad -(26)$$

where the right hand side defines the O(3) invariant Lehnert charge current density.

The lagrangian ( 24 ) can be developed as:

$$\begin{aligned}
\mathcal{L} &= D_\mu \underline{A} \cdot D^\mu \underline{A}^* + \cdots \\
&= (\partial_\mu + \underline{A}_\mu \times)\underline{A} \cdot (\partial^\mu - \underline{A}^\mu \times)\underline{A}^* + \cdots \\
&= \partial_\mu \underline{A} \cdot \partial^\mu \underline{A}^* + g \underline{A}_\mu \times \underline{A} \cdot \partial^\mu \underline{A}^* \\
&\quad - g \partial_\mu \underline{A} \cdot \underline{A}^\mu \times \underline{A}^* - g^2 (\underline{A}_\mu \times \underline{A}) \cdot (\underline{A}^\mu \times \underline{A}^*) \\
&\quad + \cdots
\end{aligned} \quad -(27)$$

and using this form in the Euler Lagrange equations ( 21 ) ~~and ( )~~

gives the O(3) invariant wave equations:

$$(\partial^\mu - g\,\underline{A}^\mu \times)((\partial_\mu + g\,\underline{A}_\mu \times)\underline{A}) = \underline{0} \quad - (28)$$

$$(\partial_\mu + g\,\underline{A}_\mu \times)((\partial^\mu - g\,A^\mu \times)\underline{A}^*) = \underline{0} \quad - (29)$$

These are different in structure from their U(1) counterparts, eqns. ( 9 )

and ( 10 ). In condensed notation { 1-3 }, eqns. ( 28 ) and ( 29 )

can be written as:

$$(\partial_\mu - ig\,A_\mu)(\partial^\mu + ig\,A^\mu)\,A = 0 \quad - (30)$$

$$(\partial_\mu + ig\,A_\mu)(\partial^\mu - ig\,A^\mu)\,A^* = 0 \quad - (31)$$

and developed as:

$$(\partial_\mu\partial^\mu + ig\,A_\mu\partial^\mu - ig\,\partial_\mu A^\mu + g^2 A_\mu A^\mu)\,A^* = 0 \quad - (32)$$

$$(\partial_\mu\partial^\mu - ig\,A_\mu\partial^\mu + ig\,\partial_\mu A^\mu + g^2 A_\mu A^\mu)\,A = 0 \quad - (33)$$

Using the quantum ansatz

$$p_\mu = i\hbar\,\partial_\mu \quad - (34)$$

eqns. ( 32 ) and ( 33 ) simplify to:

$$(\Box + g^2 A_\mu A^\mu) A^* = 0 \qquad -(35)$$

$$(\Box + g^2 A_\mu A^\mu) A = 0 \qquad -(36)$$

which have the form of Proca equations. Using eqn. ( 3 ) the O(3) invariant Proca equations are:

$$(\Box + \kappa^2) \underline{A}^* = \underline{0} \qquad -(37)$$

$$(\Box + \kappa^2) \underline{A} = \underline{0} . \qquad -(38)$$

Finally, the de Broglie Guidance Theorem:

$$\hbar \omega = m_0 c^2 \qquad -(39)$$

gives the O(3) Proca equations in the form:

$$(\Box + (m_0 c / \hbar)^2) \underline{A}^* = \underline{0} \qquad -(40)$$

$$(\Box + (m_0 c / \hbar)^2) \underline{A} = 0 \qquad -(41)$$

where $m_0$ is the rest mass of the photon.

## 3. ACQUISITION OF PHOTON MASS FROM THE VACUUM.

It is shown in this section that the introduction of a Higgs mechanism (spontaneous symmetry breaking of the vacuum) produces vacuum charge current densities in addition to the Lehnert type, which as we have seen in section (2), is produced by a local gauge transformation. In a U(1) invariant theory the Higgs mechanism is introduced through the globally invariant lagrangian:

$$\mathcal{L} = T - V = (\partial_\mu A)(\partial^\mu A^*) - m^2 A^* A - \lambda (A^* A)^2 \qquad -(42)$$

from which is obtained

$$\frac{\partial V}{\partial A} = m^2 A^* + 2\lambda A^* (A^* A). \qquad -(43)$$

If $m^2 < 0$ there is a local maximum at $A = 0$ and a minimum at

$$a^2 \equiv |A|^2 = -\frac{m^2}{2\lambda}. \qquad -(44)$$

The scalar fields $A$ and $A^*$ therefore become:

$$A(x^\mu) = a + \frac{1}{\sqrt{2}}(A_1 + iA_2) \qquad -(45)$$

$$A^*(x^\mu) = a + \frac{1}{\sqrt{2}}(A_1 - iA_2) \qquad -(46)$$

and the lagrangian ( 42 ) becomes:

$$\mathcal{L} = \partial_\mu (a + A)\partial^\mu (a + A^*) - m^2 (a + A^*)(a + A) - \lambda ((a + A^*)(a + A))^2 \qquad -(47)$$

which can be developed as:

$$\mathcal{L} = \partial_\mu A \partial^\mu A^* - 2\lambda a^2 A_1^2 - \sqrt{2} A_1 (A_1^2 + A_2^2) - \frac{\lambda}{4}\left(A_1^2 + A_2^2\right)^2 - 3\lambda a^4 . \quad -(48)$$

The Higgs mechanism has therefore acted in such a way as to produce a globally invariant field component $A_1$ with mass.

A local gauge transformation of the lagrangian ( 47 ) produces the locally invariant lagrangian:

$$\mathcal{L} = (\partial_\mu + igA_\mu)(a + A)(\partial^\mu - igA^\mu)(a + A^*) - m^2 (a + A)(a + A^*) - \lambda (a + A)^2 (a + A^*)^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \quad -(49)$$

which when used in eqn. ( 12 ) produces the field equation:

$$\partial_\nu F^{\mu\nu} = -ig\left(A^* D^\mu A - A D^\mu A^*\right) - \frac{g^2 m^2}{\lambda} A^\mu + 2\sqrt{2} g^2 a A_1 A^\mu + \sqrt{2} a g \partial^\mu A_2 . \quad -(50)$$

The term $-\frac{g^2 m^2 A^\mu}{\lambda}$ implies that the electromagnetic four potential has acquired mass in a U(1) invariant gauge theory. All four vacuum charge current densities produce vacuum energy through the equation:

$$En(vac) = \int J^\mu (vac) A_\mu \, dV \quad -(51)$$

The locally invariant lagrangian can be written out as

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + g^2 a^2 A_\mu A^\mu + \frac{1}{2} (\partial_\mu A_1)^2 + \frac{1}{2} (\partial_\mu A_2)^2 - 2\lambda a^2 A_1^2 + \sqrt{2}\, g a A^\mu \partial_\mu A_2 + \cdots \quad -(52)$$

and at its minimum value simplifies to:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + g^2 a^2 A_\mu A^\mu. \quad -(53)$$

If the mass of the photon is defined by:

$$m_0^2 := \frac{1}{2} g^2 |a^2| \quad -(54)$$

then the lagrangian ( 53 ) is a U(1) and locally invariant lagrangian for

the Proca equation:

$$\partial_\mu F^{\mu\nu} + m_0^2 A^\nu = 0. \quad -(55)$$

Therefore the photon in this U(1) invariant theory has picked up mass

from the vacuum using a Higgs mechanism.

In an O(3) invariant theory the starting point is the globally invariant lagrangian:

$$\mathcal{L} = \partial_\mu \underline{A} \cdot \partial^\mu \underline{A}^* - m^2 \underline{A} \cdot \underline{A}^* - \lambda \left( \underline{A} \cdot \underline{A}^* \right)^2 \qquad - (56)$$

from which we obtain:

$$\frac{\partial \mathcal{L}}{\partial \underline{A}} = -m^2 \underline{A}^* - 2\lambda \underline{A}^* \left( \underline{A} \cdot \underline{A}^* \right) \qquad - (57)$$

$$\frac{\partial \mathcal{L}}{\partial \underline{A}^*} = -m^2 \underline{A} - 2\lambda \underline{A} \left( \underline{A} \cdot \underline{A}^* \right) \qquad - (58)$$

from the Euler Lagrange equations ( 21 ) ~~and ( )~~. At the Higgs minimum (the symmetry broken vacuum):

$$\underline{A} \cdot \underline{A}^* = -\frac{m^2}{2\lambda} \equiv a_0^2 \qquad - (59)$$

and the wave equations ( 57 ) and ( 58 ) reduce to:

$$\Box \underline{A} = \underline{0}. \qquad - (60)$$

The locally invariant lagrangian obtained from eqn. ( 56 ) is:

$$\mathcal{L} = D_\mu \underline{A} \cdot D^\mu \underline{A}^* - \frac{1}{4} \underline{G}_{\mu\nu} \cdot \underline{G}^{\mu\nu} - m^2 \underline{A} \cdot \underline{A}^* - \lambda \left( \underline{A} \cdot \underline{A}^* \right)^2 \qquad - (61)$$

where it is understood that:

$$\underline{A} \rightarrow \underline{a}_0 + \underline{A} \qquad - (62)$$
$$\underline{A}^* \rightarrow a_0^* + \underline{A}^*$$

The Euler Lagrange equation ( 25 ) gives the field equation:

$$D_\nu \underline{G}^{\mu\nu} = -g D^\mu \underline{A}^* \times \underline{A} \qquad - (63)$$

with the Lehnert charge current density on the right hand side. At the Higgs minimum this charge current density is obtained from the symmetry broken vacuum and takes the form:

$$D_\nu \underline{G}^{\mu\nu} = -g^2 \underline{a}_0 \times (\underline{A}^\mu \times \underline{a}_0^*) \qquad - (64)$$

which is an O(3) invariant Proca equation corresponding to the lagrangian:

$$\mathcal{L} = -\frac{1}{4} \underline{G}_{\mu\nu} \cdot \underline{G}^{\mu\nu} - g^2 (\underline{A}_\mu \times \underline{a}_0) \cdot (\underline{A}^\mu \times \underline{a}_0^*) \qquad - (65)$$

The mass of the photon in the O(3) invariant theory is derived from the Higgs vacuum, which is the minimum of the potential energy used in the lagrangian ( 56 ). The field equation ( 64 ) and lagrangian ( 65 ) are O(3) invariant and so the existence of photon mass becomes compatible with the existence of the Evans Vigier field $\underline{B}^{(3)}$ {7 – 12} and the O(3) invariant B Cyclic Theorem. The Higgs mechanism is the basis of much of contemporary elementary particle theory, and this derivation is based on a rigorously O(3) and locally invariant gauge theory.

DISCUSSION

It has been shown in this paper that an O(3) invariant Proca equation can be obtained from a local O(3) invariant gauge transformation (eqns. (20) to (41)) In this mechanism the photon mass appears from the O(3) invariant local gauge transformation itself. This does not occur in a U(1) invariant gauge theory applied to electrodynamics. It has also ben shown that a Higgs mechanism applied in both a U(1) and O(3) invariant theory produces a gauge invariant Proca equation. In this mechanism the photon mass is picked up from the symmetry broken vacuum defined by the Higgs minimum.

The received view of the Proca equation { 1 - 3 } is that it is not invariant under local gauge transformation, but in this paper it has been shown that the received view is incorrect. Photon mass is compatible with rigorous U(1) and O(3) invariant gauge theory applied to electrodynamics in the vacuum. The same procedures produce self-consistently the Lehnert charge current densities in the vacuum, and its concomitant inherent energy, defined by eqn. ( 51 ). Therefore there is energy inherent in the vacuum both in a U(1) and O(3) invariant gauge theory.

The rigorous application of gauge theory to any problem requires an internal gauge space, and this is also true of gauge theory applied to electrodynamics in the vacuum. The internal gauge space has been defined in this paper through complex scalar (U(1)) and vector (O(3)) fields with a concomitant topological charge {7 – 12} defined by eqn. ( 3 ) in both cases. If the existence of this internal space is neglected, the Lehnert vacuum charge current density disappears.

It is now known that the advantages of using an O(3) invariant gauge theory applied to vacuum electrodynamics ("O(3) electrodynamics") are overwhelming { 7 – 12 }. For example interferometric effects are described with precision, one prominent example being the Sagnac effect { 13 }, another example being Michelson interferometry { 14 }. A U(1) invariant gauge theory applied to vacuum electrodynamics ("U(1) electrodynamics") fails to describe either effect {13, 14}, and fails to describe physical optics in general. O(3) electrodynamics is homomorphic with Barrett's SU(2) electrodynamics { 15 }, and both have been tested extensively against empirical data {7 - 15 }. The phase factor in both O(3) and SU(2) electrodynamics is a Wu Yang phase factor, which is related to the Evans Vigier field $\underline{B}^{(3)}$ {7 - 12} using a non-Abelian Stokes Theorem. It has been shown {12 - 14} that all interferometric effects are topological in nature, and defined through the topological charge ( 3 }. In contrast U(1) electrodynamics fails to describe the Sagnac effect because its phase is invariant under motion reversal symmetry, which generates the anticlockwise from the clockwise loop in the Sagnac effect with platform at rest { 13 }. U(1) electrodynamics fails to describe Michelson interferometry because its phase factor is invariant under parity inversion symmetry, which is equivalent to normal reflection { 12 }. O(3) electrodynamics explains Michelson interferometry and reflection through a Wu Yang phase factor { 12 }, and is also successful in explaining topological effects such as the rotation of the plane of linearly polarized light propagating through a helix { 16 }. U(1) electrodynamics has no explanation for this effect. The Aharonov Bohm effect can be explained by O(3) electrodynamics {12, 17}, whereas U(1) electrodynamics fails to give a satisfactory effect. The above mentioned is a selection of many effects {7 - 17} which O(3) explains but U(1) does not.

## ACKNOWLEDGEMENTS

Funding for individual member laboratories of AIAS is gratefully acknowledged, and the U.S. Department of Energy is thanked for website;

http://www.ott.doe.gov/electromagnetic/

## REFERENCES

{1} G. Nash and S. Sen, "Topology and Geometry for Physicists" (Academic, London, 1983).

{2} L. O'Raigheartaigh, Rep. Prog. Phys., 42, 159 (1979).

{3} L. H. Ryder, "Quantum Field Theory" (Cambridge, 1987, 2nd. Ed.).

{4} B. Lehnert, Optik, 99, 113 (1995).

{5} B. Lehnert, Phys. Scripta, 59, 204 (1996).

{6} B. Lehnert and S. Roy, "Extended Electromagnetic Theory" (World Scientific, Singapore, 1998).

{7} M. W. Evans, "The Enigmatic Photon, Volume Five" (Kluwer, Dordrecht, 1999).

{8} M. W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the $\underline{B}^{(3)}$ Field." (World Scientific, Singapore, 2000).

{9} M. W. Evans, Found. Phys., 24, 1519 (1994).

{10} M. W. Evans and S. Kielich (eds.), "Modern Non-Linear Optics", a special topical issue in three parts of I. Prigogine and S. A. Rice, (series eds.), "Advances in Chemical Physics" (Wiley, New York, 1992, 1993, 1997).

{11}M. W. Evans, J.-P. Vigier, S. Roy and S. Jeffers, "The Enigmatic Photon" (Kluwer, Dordrecht, 1994 to 1998), vols. 1 to 4.

{12} M. W. Evans (ed.), "Contemporary Optics and Electrodynamics", a special topical issue in three parts of I. Prigogine and S. A. Rice (eds.), "Advances in Chemical Physics" (Wiley, New York, 2001, in prep.), vol 114, second edition of ref. (10), reviews in vol. 114(2) and 114(3).

{13}M. W. Evans et al., AIAS group paper, Phys. Scripta, 61, 79 (2000).

{14} M. W. Evans et al., AIAS group paper, Found. Phys. Lett., 12, 579 (1999).

{15} T. W. Barrett in A. Lakhtakia (ed.), "Essays on the Formal Aspects of Electromagnetic Theory" (World Scientific, Singapore, 1993); T. W. Barrett in T. W. Barrett and D. M. Grimes (eds.), "Advanced Electromagnetism", (World Scientific, Singapore, 1995); T. W. Barrett in M. W. Evans (ed.), "Apeiron" special issue, Spring, 2000.

{16} A. Tomita and R. Y. Chiao, Phys. Rev. Lett., 57, 937, 940 (1986).

{17} M. W. Evans, S. Jeffers and J.-P. Vigier, in pat 3 of ref. (12).