SPONTANEOUS SYMMETRY BREAKING AS THE SOURCE OF THE ELECTROMAGNETIC FIELD

by

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KEYWORDS: Topological charge current density in the vacuum; Lehnert charge current density; spontaneous symmetry breaking; U(1) and O(3) gauge theory.

ABSTRACT

The source of the electromagnetic field is shown to be spontaneous symmetry breaking of the vacuum by considering a U(1) and O(3) gauge field theory with appropriate internal symmetry spaces constructed in each case from the symmetry broken vacuum. Using this procedure a topological charge current density emerges which acts as a source for the electromagnetic field in the vacuum. Gauge theory therefore demonstrates rigorously the emergence of the Lehnert charge current density from the symmetry broken vacuum.

1 INTRODUCTION

It has been shown recently in this journal and elsewhere[1]-[5] that there are a number of inconsistencies in U(1) gauge theory applied to electrodynamics, inconsistencies which are not present in a novel O(3) gauge theory[6, 7]. The received view[8] of U(1) gauge theory applied to electrodynamics overlooks the fact that in any U(1) gauge theory there must be an internal space composed of a complex scalar field with two components[9] concomitant with a conserved charge. In this Letter the internal space is re-instated and the charge is shown to be topological in nature. The lagrangian for the internal space of the gauge theory is constructed from the symmetry broken vacuum[9], so there are no excitations present above the true vacuum in the internal gauge space. There emerges from this assumption a Lehnert[10] charge current density whose origin is the true vacuum. This charge current density acts as a source for the electromagnetic field in the vacuum, thereby removing the anomaly present in the received view[8] that an electromagnetic field in the vacuum can propagate without a source, i.e. that there can be effect without cause. The analysis is repeated using an O(3) invariant gauge theory[6, 7] applied to the electromagnetic field, and a Poynting Theorem constructed for the vacuum energy flow. There is a component in this Theorem which is a constant of integration of theoretically unlimited magnitude.

2 INTERNAL SPACE

In U(1) gauge theory applied to electrodynamics the lagrangian for the internal space is assumed to be:

$$\mathcal{L} \;=\; \partial_\mu a\, \partial^\mu a^* - m^2 a^* a - \lambda (a^* a)^2 \tag{1}$$

where aa^* is defined by the local minimum[9] of the symmetry broken vacuum::

$$aa^* \;=\; -\frac{m^2}{2\lambda} \tag{2}$$

where m is a mass parameter which can become negative and λ a parameter which pre-multiplies the self interaction term. Although aa^* is a constant, the components a and a^* can be functions of x^μ, and so functional variation of the lagrangian (1) can give non-zero results. In eqn. (1), aa^* represents the minimum value of a scalar electromagnetic field:

$$a = \frac{1}{\sqrt{2}}(a_1 + ia_2) \tag{3}$$

$$a^* = \frac{1}{\sqrt{2}}(a_1 - ia_2) \tag{4}$$

$$aa^* = \frac{1}{2}(a_1^2 + a_2^2) = -\frac{m^2}{2\lambda} \tag{5}$$

in the internal space of U(1) gauge theory applied to electrodynamics in the vacuum. The complex valued scalar field is associated with a topological charge g which appears in the covariant derivative when the lagrangian (1) is subjected to a local gauge transformation. The Euler Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial a} = \partial_\mu \Big(\frac{\partial \mathcal{L}}{\partial(\partial_\mu a)} \Big) \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial a^*} = \partial_\mu \Big(\frac{\partial \mathcal{L}}{\partial(\partial_\mu a^*)} \Big) \tag{7}$$

give the globally invariant vacuum wave equations:

$$\Box\, a^* = -\big(m^2 + 2\lambda a^* a\big) a^* = 0 \tag{8}$$

$$\Box\, a = -\big(m^2 + 2\lambda a a^*\big) a = 0 \tag{9}$$

Under a local gauge transformation the lagrangian (1) becomes[6, 7, 9]:

$$\mathcal{L} = \big(\partial_\mu + igA_\mu\big) a \,\big(\partial^\mu - igA^\mu\big) a^* - m^2 aa^* - \lambda(aa^*)^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \tag{10}$$

where g is the topological charge defined by.

$$g = \frac{\kappa}{A^{(0)}} \tag{11}$$

where κ is the wavenumber and $A^{(0)}$ the scalar magnitude of the four potential A^μ which defines the electromagnetic field $F^{\mu\nu}$.

Note that the kinetic energy term $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ is not part of the gauge transformation and cannot be derived therefrom. However it gives the correct field eqn. (13) from the Euler Lagrange equation (14). Similarly, the term $\frac{1}{4}\mathbf{G}_{\mu\nu} \cdot \mathbf{G}^{\mu\nu}$ in eqn. (25) cannot be derived from a gauge transformation, but gives the correct form of the field equation.

Using this locally invariant lagrangian in eqns. (6) to (7) produces the locally invariant vacuum wave and field equations:

$$D_\mu(D^\mu a^*) \;=\; D_\mu(D^\mu a) \;=\; 0 \tag{12}$$

$$\partial_\mu F^{\mu\nu} \;=\; -igc\big(a^* D^\mu a - a D^\mu a^*\big) \tag{13}$$

from:

$$\frac{\partial\,\mathcal{L}}{\partial\,A_\nu} \;=\; \partial_\mu\Big(\frac{\partial\,\mathcal{L}}{\partial(\partial_\nu A_\mu)}\Big) \tag{14}$$

Eqn. (13) is an inhomogeneous field equation which contains a vacuum charge current density term on the right hand side. So a rigorous application of U(1) gauge theory to vacuum electrodynamics gives the Lehnert charge current density from the symmetry broken vacuum, i.e. from the ground state of the field theory. Both the left and right hand sides of eqn.(13) are defined by the minimum of potential energy, and by the minimum value that AA^* can attain in the symmetry broken vacuum. This minimum value is aa^*, and is the expectation value of AA^*. Therefore in this view the source of the electromagnetic field propagating in the vacuum is determined by this minimum value.

The lagrangian (10) can be written as:

$$\mathcal{L} \;=\; g^2 a^2 A_\mu A^\mu + \cdots \tag{15}$$

and contains a term that can be identified with the mass of the photon:

$$m_p^2 \;=\; 2g^2\big|a^2\big| \tag{16}$$

giving the locally gauge invariant Proca equation[6, 7]:

$$\partial_\nu F^{\mu\nu} + m_p^2 A^\mu \;=\; 0 \tag{17}$$

Therefore application of the Higgs mechanism in this way has produced one massive photon from one massless photon. The scalar fields a and a^* do not acquire mass, and degrees of freedom are conserved. In this view therefore, photon mass is identified to be the result of a local gauge transformation applied at the minimum of the symmetry broken vacuum, i.e. the minimum value that the potential energy of the globally invariant lagrangian can attain.

The minimum value gives the true vacuum energy:

$$En(vac) \;=\; \int J^\mu(vac) A_\mu dV \tag{18}$$

and a rate of doing work:

$$\frac{dW(vac)}{dt} \;=\; \int \mathbf{J}^\mu(vac)\cdot\mathbf{E}_\mu\, dV \tag{19}$$

so a Poynting Theorem can be developed for the vacuum. This is detailed later in this Letter.

It has been shown by several authors[1]-[7],[11]-[14] that an O(3) invariant gauge field theory of vacuum electrodynamics has many advantages over a U(1) invariant gauge field theory. In this case, the internal space is a vector space, so the globally invariant lagrangian (1) becomes:

$$\mathcal{L} \;=\; T - V \;=\; \partial_\mu \mathbf{a}\cdot\partial^\mu \mathbf{a}^* - m^2 \mathbf{a}\cdot\mathbf{a}^* - \lambda\big(\mathbf{a}\cdot\mathbf{a}^*\big)^2 \tag{20}$$

with potential energy:

$$V \;=\; m^2\mathbf{a}\cdot\mathbf{a}^* + \lambda\big(\mathbf{a}\cdot\mathbf{a}^*\big)^2 \tag{21}$$

The equation of motion is:

$$\frac{\partial\,\mathcal{L}}{\partial\mathbf{a}} \;=\; \partial_\nu\Big(\frac{\partial\,\mathcal{L}}{\partial_\nu\mathbf{a}}\Big); \qquad \frac{\partial\,\mathcal{L}}{\partial\mathbf{a}^*} \;=\; \partial_\nu\Big(\frac{\partial\,\mathcal{L}}{\partial_\nu\mathbf{a}^*}\Big) \tag{22}$$

and produces the globally invariant wave equation:

$$\Box\,\mathbf{a}^* \;=\; \mathbf{0} \tag{23}$$

$$\Box\,\mathbf{a} \;\;=\; \mathbf{0} \tag{24}$$

Local gauge transformation of the lagrangian (20) produces:

$$\mathcal{L} \;=\; D_\mu\mathbf{a}\cdot D^\mu\mathbf{a}^* - \frac{1}{4}\mathbf{G}_{\mu\nu}\cdot\mathbf{G}^{\mu\nu} - m^2\mathbf{a}\cdot\mathbf{a}^* - \lambda\big(\mathbf{a}\cdot\mathbf{a}^*\big)^2 \tag{25}$$

with eqn.(22) produces the wave equations:

$$\begin{aligned} \big(\partial^\mu - g\mathbf{A}^\mu\times\big)\Big(\big(\partial_\mu + g\mathbf{A}_\mu\times\big)\mathbf{a}\;\Big) &= \mathbf{0} \\ \big(\partial_\mu + g\mathbf{A}_\mu\times\big)\Big(\big(\partial^\mu - g\mathbf{A}^\mu\times\big)\mathbf{a}^*\Big) &= \mathbf{0} \end{aligned} \tag{26}$$

The Euler Lagrange equation:

$$\frac{\partial\,\mathcal{L}}{\partial\,\mathbf{A}_\mu} \;=\; \partial_\nu\Big(\frac{\partial\,\mathcal{L}}{\partial(\partial_\nu\mathbf{A}_\mu)}\Big) \tag{27}$$

produces the O(3) invariant field equation:

$$D_\nu\mathbf{G}^{\mu\nu} \;=\; -g^2\mathbf{a}\times\big(\mathbf{A}^\mu\times\mathbf{a}^*\big) \tag{28}$$

where the current on the right hand side is generated by the minimum value of $\mathbf{A}$ in the internal O(3) symmetry gauge space. This minimum value is the vacuum and is denoted by the vector $\mathbf{a}$.

The lagrangian (25) can be written as:

$$\mathcal{L} \;=\; -g^2\mathbf{A}_\mu\times\mathbf{a}\cdot\mathbf{A}^\mu\times\mathbf{a}^* + \cdots \tag{29}$$

and produces three photons with mass from the vector identity:

$$\big(\mathbf{A}_\mu \times \mathbf{a}\big) \cdot \big(\mathbf{A}^\mu \times \mathbf{a}^*\big) \;=\; \big(\mathbf{A}_\mu \cdot \mathbf{A}^\mu\big)\big(\mathbf{a} \cdot \mathbf{a}^*\big) - \big(\mathbf{A}_\mu \cdot \mathbf{a}\big)\big(\mathbf{a}^* \cdot \mathbf{A}^\mu\big) \tag{30}$$

and the term:

$$\mathcal{L} \;=\; -g^2\big(\mathbf{a} \cdot \mathbf{a}^*\big)\big(\mathbf{A}_\mu \cdot \mathbf{A}^\mu\big) \tag{31}$$

One of these is the super-heavy Crowell boson, recently observed[15] empirically, and associated with index (3) in the $((1),(2),(3))$ basis, and the other two are massive photons associated with indices (1) and (2)[6, 7]. In an O(3) invariant gauge theory the vacuum current in S.I. units is:

$$\mathbf{J}^\mu(vac) \;=\; -\frac{g}{\mu_0 c}\big(D^\mu \mathbf{a}^*\big) \times \mathbf{a} \tag{32}$$

and the vacuum energy is:

$$En \;=\; \int \mathbf{J}^\mu(vac) \cdot \mathbf{A}_\mu dV \tag{33}$$

This can be transformed into a matter current by a minimal prescription:

$$\frac{\kappa}{A^{(0)}} \;=\; \frac{e}{\hbar} \tag{34}$$

This matter current, where $-e$ is the charge on the electron, is effectively an electromotive force in a circuit, originating in the topological charge g of the vacuum electromagnetic field.

3 VACUUM POYNTING THEOREM

The vacuum property $J^\mu(vac)$ in a U(1) invariant gauge theory applied to electrodynamics generates an energy:

$$En \;=\; \int \mathbf{J}^\mu(vac) \cdot \mathbf{A}_\mu dV \tag{35}$$

and a rate of doing work:

$$\frac{dW}{dt} \;=\; \int \mathbf{J}(vac) \cdot \mathbf{E}\, dV \tag{36}$$

The volume V is arbitrary and from eqn. (36) standard methods give a vacuum Poynting Theorem:

$$\frac{dU(vac)}{dt} + \nabla \cdot \mathbf{S}(vac) \;=\; -\mathbf{J}(vac) \cdot \mathbf{E} \tag{37}$$

or law of conservation of energy/momentum between vacuum and field. The vacuum Poynting vector $\mathbf{S}(vac)$ represents energy flow and is defined as:

$$\nabla \cdot \mathbf{S}(vac) \;=\; -\mathbf{J}(vac) \cdot \mathbf{E} \tag{38}$$

Integrating this equation gives:

$$\mathbf{S}(vac) \; = \; -\int \mathbf{J}(vac) \cdot \mathbf{E} \, d\mathbf{r} + \textsf{constant of integration} \tag{39}$$

where the constant of integration represents a physical component of energy flow whose magnitude can become arbitrarily large. This is the Heaviside component of energy flow neglected in the received view[8] but is theoretically a source of energy flow when U(1) gauge theory is rigorously applied to electrodynamics. The physical object also emanates from the vacuum and its magnitude is not limited by any constraint of gauge theory, because A^μ is not limited in magnitude. The energy flow represented by $\mathbf{S}(vac)$ is electromagnetic energy flow, and can be converted in principle to mechanical, or thermal, energy with available devices[3] which give a theoretically unlimited source of energy.

The physical meaning of the vacuum Poynting Theorem is that the time rate of change of electromagnetic energy within an arbitrary volume V, combined with energy flow out through the boundary surfaces of the arbitrary volume per unit time is equal to the negative of the total work done by the field on the source, interpreted as vacuum charge current density as developed earlier in this Letter. This is a statement of conservation of energy applied within the vacuum and in the absence of matter (electrons).

In the received view:

$$J^\mu(vac) \; = \; 0 \tag{40}$$

and there is no vacuum Poynting Theorem. In the correctly gauge invariant equation (37), work is done by the source (the vacuum) on the field, work which can be transmitted to rate of change of mechanical energy as follows[8]:

$$\frac{dEn(mech)}{dt} \; = \; \int \mathbf{J}(vac) \cdot \mathbf{E} \, dV \tag{41}$$

Therefore there is support in basic gauge theory (properly applied) for working devices[3] which produce energy from the vacuum.

ACKNOWLEGEMENTS

Funding for individual member laboratories of AIAS is acknowledged and funding from the U. S. Department of Energy is acknowledged both for supercomputer time and for the website:

http://www.ott.doe.gov/electromagnetic/

Various other sources of funding are also acknowledged with gratitude.

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