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Unification of Gravitation and Electromagnetism with $\mathbf{B}^{(3)}$

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The experimentally supported existence of the Evans–Vigier field, $\mathbf{B}^{(3)}$, in vacuo implies that the gravitational and electromagnetic fields can be unified within the same Ricci tensor, being respectively its symmetric and antisymmetric components in vacuo. The fundamental equations of motion of vacuum electromagnetism are developed in this framework.

1. INTRODUCTION

In this paper a unified view is developed of the vacuum gravitational and electromagnetic fields using the recent emergence of the B cyclic equations,[1–12] which, for the vacuum electromagnetic field, relates the transverse plane waves of magnetic flux density, $\mathbf{B}^{(1)} = \mathbf{B}^{(2)*}$, to the novel and longitudinal $\mathbf{B}^{(3)}$ field,

$$\mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = iB^{(0)}\mathbf{B}^{(3)*} \text{ et cyclicum} \tag{1}$$

Here $B^{(0)}$ is the scalar magnitude of $\mathbf{B}^{(1)}$, $\mathbf{B}^{(2)}$, and $\mathbf{B}^{(3)}$, and the complex basis [(1), (2), (3)] has been used.[13, 14] The latter is a convenient complex representation of three-dimensional space which is equivalent to the real, Cartesian basis. Equations (1) (and related[15] E, A, and R cyclics) are non-linear and non-Abelian, and show that $iB^{(0)}\mathbf{B}^{(3)*}$ is an experimental

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observable in magneto-optics,[16] being equal to the well-known conjugate product $\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}$. Therefore $\mathbf{B}^{(3)}$ is a real and physical component which implies that the vacuum electromagnetic field becomes describable by a Poincaré group symmetry.[16, 17] Magnetic components become proportional to rotation generators, electric components to boost generators. The field strength tensor becomes the Evans–Vigier $G_{\mu\nu}$, which contains within it the $\mathbf{B}^{(3)}$ field as an antisymmetric conjugate product of the transverse plane wave components of the vector potentials in vacuo,

$$\mathbf{B}^{(3)*} = -i \frac{e}{\hbar} \mathbf{A}^{(1)} \times \mathbf{A}^{(2)} \tag{2}$$

where $e/\hbar$ is a universal constant, the ratio of the quanta of charge and action. Thus, quantization of the electromagnetic field in this view takes place through the minimal prescription

$$\hbar\kappa = eA^{(0)} \tag{3}$$

for the free photon. Here $A^{(0)}$ is the magnitude of $\mathbf{A}^{(1)} = \mathbf{A}^{(2)}$, and κ the magnitude of the wave-vector. Therefore $eA^{(0)}$ is the free photon's linear momentum in vacuo.

In Sec. 2, the theory of the electromagnetic field in vacuo is developed through a generalization of Eq. (3), in which the $G_{\mu\nu}$ tensor of Evans and Vigier is directly proportional to a novel antisymmetric Ricci tensor, $R_{\mu\nu}^{(A)}$, obtained by suitable contraction of the Riemann tensor $R^{\kappa}_{\lambda\mu\nu}$, the same measure of curvature as in the theory of gravitation. Section 3 gives consideration to the metric, Lorentz equation, and other aspects of the geometrical theory of electromagnetism in vacuo akin to the general relativistic theory of gravitation.

2. THE EQUATIONS OF MOTION

The theory of the unified gravitational and electromagnetic fields is based[18, 19] on a novel antisymmetric Ricci tensor $R_{\mu\nu}^{(A)}$ obtained by contraction from the Riemann curvature tensor $R^{\kappa}_{\lambda\mu\nu}$. By setting $\kappa = \lambda$ we recover

$$R_{\mu\nu}^{(A)} := R^{\lambda}_{\lambda\mu\nu} \tag{4}$$

Electromagnetism is the field defined by the minimal prescription,[18, 19]

$$\hbar R_{\mu\nu}^{(A)} = eG_{\mu\nu} \tag{5}$$

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where $G_{\mu\nu}$ is the field-strength tensor introduced by Evans and Vigier following the discovery of the $\mathbf{B}^{(3)}$ field and B cyclics.[1–12] From Eqs. (4) and (5),

$$G^{\lambda}_{\lambda\mu\nu} := G_{\mu\nu} = \frac{\hbar}{e} R^{\lambda}_{\lambda}\mu\nu \tag{6}$$

$R^{\lambda}{}_{\lambda\mu\nu}$

Since $e/\hbar$ is a universal constant, the electromagnetic field-strength tensor becomes the antisymmetric part of the Riemann tensor itself. Unification with the gravitational field is achieved because the symmetric part of the *same* Riemann curvature tensor is given by the Einstein equation,[20, 21]

$$R^{(S)}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi k}{c^4} T^{(S)}_{\mu\nu} \tag{7}$$

where $T^{(S)}_{\mu\nu}$ is the canonical energy-momentum density (a symmetric tensor), and k and c are respectively the Einstein gravitational constant and the speed of light in vacuo. Therefore the vacuum electromagnetic and gravitational fields become different parts of the same fourth-rank Riemann tensor that describes spacetime curvature in curvilinear coordinates. By comparing Eqs. (6) and (7) we obtain in vacuo[19]

$$c = \lambda f = \left(\frac{4k}{c^2 L^2}\right) \hbar \tag{8}$$

where L is the Planck length.[18, 19] Therefore the speed of light in vacuo, c, becomes quantized through $4k/(c^2 L^2)$ and the constancy of the electrodynamic property λf, the product of the wavelength and frequency, becomes expressible in terms of the gravitational constant k and the Planck length L. Equation (8) is an expression of the fact that there is a quantum of mass m which cannot be defined independently of the quantum of charge, e, and which implies that there is a minimum amount of energy, mc^2, in the rest frame of every particle. The quanta m and e are defined by singularities respectively of the gravitational and electromagnetic fields, singularities which occur because of the curvature of spacetime, the curvilinear coordinate frame common to both fields. This local, geometrical, theory is an explanation for the existence of fields in terms of spacetime curvature, and since there is, presumably, only one curved spacetime, there is only one frame upon which to build all types of fields. The concept of *field* becomes synonymous with that of a curving, or flexible, coordinate system summarized by the Riemann tensor itself.

This is of course the accepted view of gravitation, as first proposed and developed by Einstein.[21] As described by Misner *et al.*[20]:

> "Space acts on matter, telling it how to move. In turn, matter reacts back on space, telling it how to curve.... To produce curvature in space here is to force curvature in space there.... Thus matter here influences matter there.... In Newtonian theory, this effect is ascribed to gravitation acting at a distance from the center of attraction. According to Einstein, a particle gets its moving orders locally, from the geometry of spacetime—to follow the straightest possible track (geodesic). Physics is as simple as it could be locally.... The effect of geometry on matter causes originally divergent geodesics to cross, and the effect of matter on geometry is bending of spacetime initiated by a concentration of mass."

These well-known and accepted statements abstracted from Misner *et al.*[20] can now be adapted for electrodynamics as follows. Curvature in space is produced not only by mass, but also by the charge quantum, e; and this forces curvature which tells another charge how to move. This view replaces the early nineteenth century action at a distance between charges, and geometry becomes synonymous with the electromagnetic field, which, as for the gravitational field, becomes curvature of spacetime itself. However, the Einstein tensor (symmetric combination of Ricci tensors, $R^{(S)}_{\mu\nu} - (1/2)\, g_{\mu\nu} R$) is replaced for electromagnetism by a novel antisymmetric counterpart, the Ricci tensor $R^{(A)}_{\mu\nu}$.[18, 19] The unified gravitational and electromagnetic field is described in terms of components of the *same* overall Ricci tensor, respectively symmetric and antisymmetric. The warping of spacetime due to $R^{(A)}_{\mu\nu}$ is the effect of the charge quantum on geometry, while the warping of spacetime due ta the Einstein tensor is the effect of the mass quantum on geometry. In our unified theory of fields, gravity and electromagnetism are different parts of the Ricci tensor $R_{\mu\nu}$, and therefore different parts of the Riemann curvature tensor itself, i.e., are different aspects of the same overall spacetime geometry. The link between charge and mass is established at the most fundamental level and in its simplest form by the scalar relation[1–12]

to

$$p = \hbar\kappa = eA^{(0)} \tag{9}$$

between photon momentum magnitude $\hbar\kappa$ and the momentum magnitude $eA^{(0)}$, the result of the minimal prescription applied to the free photon-field. Here A is the scalar magnitude of the complex plane-wave vector potential[1–12] $\mathbf{A}^{(1)} = \mathbf{A}^{(2)*}$, and κ is the wavenumber. Equation (9) shows that the momentum of the particulate photon is the product of two $\hat{C}$ negative[1, 5] quantities, e and $A^{(0)}$, proportional therefore to the product of two elementary charges, and therefore *quadratic* in e. The minimal prescription (9) shows that momentum is proportional to charge squared, but is linear in mass if nonzero. The $\hat{C}$ symmetry of momentum must be positive[1, 5] in the minimal prescription, in which derivatives are replaced by covariant derivatives, implying curved spacetime. If the electromagnetic field is considered to be radiated by electrons, then e is negative, if by positrons, e is positive. In *both* cases, however, momentum is positive.

Equation (5) is the counterpart of Eq. (9) in general relativity, i.e.,

$$\boxed{\hbar\kappa = eA^{(0)}} \quad \rightarrow \quad \boxed{\hbar R_{\mu\nu}^{(A)} = eG_{\mu\nu}} \tag{9a}$$

so that $A^{(0)}$ is proportional to the magnitude of $G_{\mu\nu}$. The photon momentum itself can therefore be interpreted as a curvature of spacetime, and the minimal prescription asserts that the momentum, $\hbar R_{\mu\nu}^{(A)}$, is the product of e with the field strength tensor $G_{\mu\nu}$ of electromagnetic waves in vacuo. Equation (5) consistently accounts for the presence of covariant derivatives, because the Maxwellian $F_{\mu\nu}$ tensor[23] has been replaced by the $G_{\mu\nu}$ tensor[24] in which the $\mathbf{B}^{(3)}$ field is incorporated self-consistently as[2]

$$\mathbf{B}^{(3)*} = -i\frac{e}{\hbar}\mathbf{A}^{(1)} \times \mathbf{A}^{(2)} \tag{10}$$

in the basis ((1), (2), (3)). Equation (10) is a member of a set of equations with 0(3) space symmetry, or Poincaré group spacetime symmetry, and shows that the $\mathbf{B}^{(3)}$ field is an invariant of special relativity. It propagates through the vacuum at c, F.A.P.P., and accompanies the plane waves as described in Sec. 1.

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These inferences are missing completely from Maxwellian electrodynamics in vacuo, which is an incomplete field theory valid only in flat spacetime, and which therefore uses partial derivatives ∂_μ to describe the free space propagation of the electromagnetic field instead of the Lorentz covariant derivatives D_μ needed in curved spacetime, and which are inbuilt to general relativity. In a hypothetical universe built on flat spacetime, the Riemann curvature tensor and Ricci tensor vanish, meaning that an equation such as (5) becomes meaningless. If there is no spacetime curvature, $R_{\mu\nu}^{(A)} = R_{\mu\nu}^{\lambda}$ is zero, and therefore so is $G_{\mu\nu}$. In consequence, there is no electromagnetic field present, because the latter is now understood to be due to curvature of spacetime.

$R^{\lambda}_{\lambda\mu\nu}$

There is a basic difference therefore between our geometrical theory of electromagnetism expressed in Eq. (5) and the Maxwellian theory, from which Riemannian geometry is missing, and which has no concept of curved spacetime. This is clear from the fact that Maxwellian electrodynamics is the basis of special, not general, relativity. *Experimental* magneto-optics show, however,[3, 25] that circularly polarized electromagnetic radiation produces magnetization phenomena of various kinds which force the Maxwellian theory to construct phenomenologically[26] the conjugate product $\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}$ in vacuo. Without this construct of nonlinear optics[27] there appears to be no method by which Maxwellian electrodynamics can account for these observable effects, of which there are several now known.[13, 25, 28–34] By using $\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}$ in vacuo, however, *Maxwellian theory becomes deeply inconsistent within itself*, something that was first noticed in 1992.[4] There exists in vacuo the *observable* quantity $iB^{(0)}\mathbf{B}^{(3)*} = \mathbf{B}^{(1)} \times \mathbf{B}^{(2)}$ (Sec. 1). This represents a fatal flaw in the Maxwellian view which comes to sight only in the tiny and difficult-to-observe effects of magneto-optics,[13, 28–34] a flaw which has ramifications throughout contemporary field theory. The $\mathbf{B}^{(3)}$ (Evans–Vigier) field exists in Vacuo and is real and experimentally observable because $iB^{(0)}\mathbf{B}^{(3)*}$ is an observable. In the Maxwellian theory there can be only transverse waves $\mathbf{B}^{(1)} = \mathbf{B}^{(2)*}$, and the very existence of the longitudinally polarized $\mathbf{B}^{(3)}$ in vacuo becomes a fundamental paradox. Developments[18, 19] leading to Eq. (5) show that the root cause of the paradox is the implicit assumption of Maxwellian theory (i.e., of special relativity) that spacetime is flat, and remains flat even when the electromagnetic field interacts with material matter, represented by a fermion, for example, in Dirac's equation. This is a view that blatantly contradicts that of general relativity, in which spacetime is curved by matter and remains so in vacuo. Both the Maxwell and Einstein theories have worked so well for so long, however, that they have been deservedly accepted as pinnacles of thought. The tiny magneto-optic effects through which Nature shows the limits of the Maxwellian point of view were of course unknown both to Einstein and to Maxwell, having been developed in the mid sixties[1–12] to present. We suggest in this and other papers[18, 19] on $\mathbf{B}^{(3)}$ in general relativity that the paradox can be resolved through a development of equations such as (5), in which electromagnetism and gravitation are built on the same foundation.

By proceeding in this way, the equations of vacuum electrodynamics become[18, 19]

$$D_\rho G_{\mu\nu} + D_\nu G_{\rho\mu} + D_\mu G_{\nu\rho} = 0 \tag{11a}$$

$$D_\nu G^{\mu\nu} = 0 \tag{11b}$$

Equation (11a) is the Bianchi identity[20, 21] for $R^{\lambda}_{\lambda\mu\nu} = R^{(A)}_{\mu\nu}$ through use of Eq. (5). If D_μ is replaced by ∂_μ and $G_{\mu\nu}$ by $F_{\mu\nu}$, then we can force the reduction of Eq. (11a) to the homogeneous Maxwell equations in vacuo. However, this is now a paradoxical procedure, because we are replacing curved spacetime by flat spacetime, and arbitrarily eliminating $\mathbf{B}^{(3)}$ from

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consideration. So we are no longer taking Maxwellian electrodynamics as our starting point; we are starting from the geometrical Eqs. (11a) and (11b). Similarly, Eq. (11b), which follows from Eq. (5), becomes the inhomogeneous Maxwell equations in vacuo,[2] but in the same arbitrary way.

Therefore the flaw in Maxwell's theory consists in *precisely* these assumptions, that D_μ of curved spacetime can be replaced by ∂_μ, and that $G_{\mu\nu}$ can be replaced by $F_{\mu\nu}$. These replacements are synonymous with the replacement of curved with flat spacetime, because D_μ is defined in terms of the affine parameter or Christoffel symbol, which vanishes in flat spacetime. Since $\mathbf{B}^{(3)}$ is *defined* in vacuo by Eq. (10), which is an outcome of the use of $G_{\mu\nu}$[2] in vacuum electrodynamics, the replacement of $G_{\mu\nu}$ by $F_{\mu\nu}$ means that $\mathbf{B}^{(3)}$ is discarded arbitrarily from the theory. The homogeneous Maxwell equations, for example, are solved to give transverse components $\mathbf{B}^{(1)} = \mathbf{B}^{(2)*}$, and because ∂_μ replaces D_μ in Eqs. (11), the Maxwell equations are linear in field strength. Equations (11), on the other hand, are nonlinear, precisely because of the use of D_μ.[2] Equation (10) shows, however, that the nonlinear cross product of $\mathbf{A}^{(1)}$ with $\mathbf{A}^{(2)}$ produces $\mathbf{B}^{(3)*}$ through the universal constant $e/\hbar$. This property allows Eqs. (11) to be decoupled[2] and solved for $\mathbf{B}^{(1)}$, $\mathbf{B}^{(2)}$, and $\mathbf{B}^{(3)}$, which become proportional to angular momentum generators, or, within a factor $\hbar$, infinitesimal rotation generators of the Poincaré group.[1] Therefore $\mathbf{B}^{(3)}$, paradoxical in the special relativistic (Maxwellian) theory of the vacuum electromagnetic field, becomes the experimental basis of electromagnetism In general relativity.

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Furthermore, the way in which $\mathbf{B}^{(3)}$ magnetizes matter is given by Eq. (10). If the Dirac equation [3] is used to describe a fermion in a circularly polarized electromagnetic field, the conjugate product $\mathbf{A}^{(1)} \times \mathbf{A}^{(2)}$ becomes an intrinsic part of the structure of the Dirac equation itself through the intermediacy of the minimal prescription.[3] By incorporating intrinsic spin into general relativity it can be shown that $\mathbf{B}^{(3)}$ becomes expressible in terms of Fock–Ivanenko coefficients[3] and is thereby shown to be the relict magnetic field of background radiation in relativistic cosmology.

These are powerful arguments in favor of a geometrical theory of electromagnetism, and Eq. (5) seems to be a natural way of describing electromagnetic field strength in vacuo in terms of a contraction of the Riemann tensor that leads to an antisymmetric Ricci tensor. The latter is an angular momentum four-tensor[1–3] through the relation[18, 19]

$$R_{\mu\nu}^{(A)} = \frac{\kappa^2}{\hbar} J_{\mu\nu} \tag{12}$$

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so it is possible to link $R^{(A)}_{\mu\nu}$ to an antisymmetric angular energy-angular momentum tensor through

$$T^{(A)}_{\mu\nu} := \omega J_{\mu\nu} = \frac{\hbar\omega}{\kappa^2} R^{(A)}_{\mu\nu} \tag{13}$$

where ω is the angular frequency of the electromagnetic field.

Equation (13) is a type of Einstein equation [cf. Eq. (7)], from which we obtain

$$T^{(A)}_{\mu\nu} = e\frac{c}{\kappa} G_{\mu\nu} \tag{14}$$

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showing that $T^{(A)}_{\mu\nu}$ is proportional to $G_{\mu\nu}$. From the properties of the Bianchi identity it follows that

$$T^{\mu\nu(A)}_{\ \ ;\nu} = e\frac{c}{\kappa} G^{\mu\nu}_{\ \ ;\nu} = 0 \tag{15}$$

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from which

$$D_\nu G^{\mu\nu} := G^{\mu\nu}_{\ \ ;\nu} = 0 \tag{16}$$

follows in turn, confirming Eq. (11b). Therefore, in precise analogy with general relativity, the equations of motion are derived from the covariant derivative of the energy-momentum tensor.

At this point, it becomes convenient to re-examine the older point of view of gravitation vis à vis electromagnetism through a statement by Bose:[22]

> "The fact that the Einstein field equations predict the equation of motion is quite remarkable, and may be contrasted to the situation in electrodynamics, where the Maxwell equations do *not* (Bose's emphasis) contain the corresponding equation of motion. The origin of this distinction lies in the nonlinear character of the Einstein equation. The physical significance of this nonlinearity resides in the fact that the gravitational fields carry energy, while, for instance, electromagnetic fields do not carry charge."

Similar statements concerning the apparent incompatibility of the gravitational and electromagnetic fields appear in many textbooks.

However, the new geometrical theory of electromagnetism being developed in this paper and elsewhere[19] derives the basic electrodynamical equations (11) *from the Bianchi identity involving the same Riemann tensor as used in general relativity*. In gravitational theory,[20, 21] the Bianchi identity requires that the tensor $R^{(A)}_{\mu\nu} = R^{\lambda}_{\lambda\mu\nu}$ have zero covariant divergence, a requirement that leads via Eq. (7), the Einstein equation, to

$$T^{\mu\nu}_{\ \ ;\nu} = 0 \tag{17}$$

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where $T_{\mu\nu}$ is the gravitational energy-momentum density tensor. Equation (17) leads to the equations of motion which show that the position of a particle is where the field equations become singular. The field equations determine how this singularity (delta function) should move, obeying the same equation of motion as that of a test particle.

In the geometrical theory of vacuum electromagnetism we can build a precise analogy as follows. The Bianchi identity now requires that the tensor $R^{(A)}_{\mu\nu} = R^{\lambda}_{\lambda\mu\nu}$ have-zero covariant divergence, which leads immediately to Eq. (11b) via Eq. (5). Equation (11b) is the geometrical replacement of Maxwell's inhomogeneous vacuum equation $F^{\mu\nu}_{\ ,\nu} = 0$. The same requirement leads to Eq. (15). It follows that the equation of motion of charge is obtained from Eq. (5), which is, as we have seen, a geometric generalization of the minimal prescription in vacuo, i.e., for free photon momentum $\hbar\kappa$. Therefore charge enters explicitly into the equation of motion in the geometrical theory of vacuum electromagnetism. The key to these developments is the nonlinear relation between $iB^{(0)}\mathbf{B}^{(3)*}$ and the conjugate product $\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}$. The latter is an experimental observable which shows that $\mathbf{B}^{(3)}$ is also an experimental observable.

have zero

Equations (11) are themselves therefore equations of motion, because they involve e, which appears as part of the universally constant ratio of quanta $e/\hbar$ in the basic equation (5), the minimal prescription for the free photon field in vacuum electrodynamics in general relativity. The ratio $e/\hbar$ appears in the definition of $\mathbf{B}^{(3)}$ in Eq. (10), a definition which identifies $eA^{(0)}$ with the quantized photon momentum $\hbar\kappa$. This relation implies that the quantum of charge e is given in vacuo by the product of $\hbar$ and $\kappa/A^{(0)}$. It can be show[3] that Eq. (10) represents the spiralling motion of charge in vacuo, and this is identical to a description of the motion of a test charge in a circularly polarized electromagnetic field. This spiralling motion is now seen to be *the spinning of spacetime itself*, because $R^{(A)}_{\mu\nu}$ is proportional to an angular momentum tensor $J_{\mu\nu}$ [Eq. (12)] and $J_{\mu\nu}$ is obtained therefore by contraction of the Riemann tensor *itself*. What Maxwell discerned as an electromagnetic field therefore becomes a spinning of curved spacetime itself. In this sense, the charge e has no choice except to spiral, because spacetime demands that its motion be predetermined in this way and that the spiral be the geodesic for charge.

(3) shown

In the same way precisely, symmetric curvature (e.g., funnelling) of spacetime demands that point mass be guided in a predetermined fashion. Conversely, mass funnels spacetime and charge spins spacetime, both phenomena being derived from the same basic Riemann curvature tensor by different types of index contraction. Thus, the phenomena we know as electromagnetic and gravitational fields are manifestations of the same curvilinear geometry of spacetime. This is a natural conclusion because there

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is only one spacetime. In matter it appears that charge is always accompanied by mass, so a charged fermion, for example, must funnel and spin spacetime simultaneously, having both mass and charge. The funnelling and spiralling of spacetime eventually influences another charge e in another event of general relativity. There is a whirlpool of spacetime containing the fermion at its eye. Gravitational and electromagnetic waves manifest themselves as properties of warped spacetime. In the quantum theory they become gravitons and photons, which are also synonymous with spacetime itself. Charge and mass are aspects of the same thing, being ultimately singularities in a single warped spacetime. The link between them is given in its simplest form by the minimal prescription $eA^{(0)} = \hbar\kappa$. One side of this equation is momentum, the other is proportional to the square of e. The unification of charge and mass is implied by that of electromagnetic and gravitational fields in terms of geometry. At the most fundamental level, charge-mass becomes a singularity of the unified field, which is in turn a singularity of spacetime *itself*, the eye of the Whirlpool. In the last analysis therefore, mass and charge can be described as warped spacetime.

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3. THE METRIC, LORENTZ EQUATION, AND OTHER CONSIDERATIONS

The metric associated with electromagnetism in curved spacetime is antisymmetric, in contrast with the symmetric metric $g_{\mu\nu}$ used in general relativistic theories of gravitation.[20] Consider the completely covariant Riemann curvature tensor,

$$R_{\lambda\mu\nu\kappa} = g_{\lambda\alpha} R^{\alpha}_{\mu\nu\kappa} \tag{18}$$

If $\lambda = \mu = \alpha$ we obtain the dimensionless, but antisymmetric, tensor:

$$g^{(A)}_{\nu\kappa} = \frac{1}{R} R_{\lambda\lambda\nu\kappa} = \frac{g}{R} R^{(A)}_{\nu\kappa} \tag{19}$$

where $R^{(A)}_{\nu\kappa} = R^{\lambda}_{\lambda\nu\kappa}$ as usual. Here R is the scalar curvature[20] and $g = g_{\lambda\lambda}$ is gas is the trace of $g_{\lambda\alpha}$. Therefore,

$$g^{(A)}_{\nu\kappa} = \frac{e}{\hbar}\frac{g}{R} G_{\nu\kappa} \tag{20}$$

is directly proportional to $G_{\nu\kappa}$ in vacuo, and is a metric tensor for vacuum electromagnetism in general relativity. In contrast to the well-known metric

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$g_{\mu\nu}$ of gravitation it is off-diagonal. Its covariant derivative vanishes because [Eq. (11b)] that of $G_{\mu\nu}$ vanishes. By using the 4-D Levi-Civita symbol[1, 10] the metric $g_{\mu\nu}^{(A)}$ becomes an axial *four-vector*, proportional directly to the magnetic field axial four-vector,[9]

$$B_\mu = -\frac{i}{2c}\varepsilon_{\mu\nu\rho\sigma} G_{\rho\sigma} \varepsilon_\nu \tag{21}$$

Here

$$\varepsilon_\nu := (0, 0, 1, i) \tag{22}$$

is a unit four-vector along the axis of propagation of the photon field. Using methods developed elsewhere we obtain[9]

$$B_\mu = (0, 0, B_z, iB_z) \tag{23}$$

where

$$\mathbf{B}^{(3)} := B_z \mathbf{k}^{(3)} \tag{24}$$

Therefore the metric becomes, in Minkowski notation,[9]

$$g_\mu = \frac{e}{\hbar}\frac{g}{R} c(0, 0, B_z, iB_z) \tag{25}$$

showing that $\mathbf{B}^{(3)}$ is a primeval, or relict,[3] magnetic field of relativistic cosmology, because apart from a constant proportional to g/R, it *completely* defines the metric g_μ in axial four-vector form.

Equation (25) also demonstrates that vacuum electrodynamics in general relativity is a spinning of spacetime, a process which therefore takes place in a *noninertial* frame and which generates angular momentum and centripetal *acceleration*.

Without nonzero $\mathbf{B}^{(3)}$, the metric g_μ vanishes, showing that the $\mathbf{B}^{(3)}$ field is the key to the geometrical theory of electrodynamics in vacuo. By assuming only that there exists a primeval charge quantum, e, it can be shown that $\mathbf{B}^{(3)}$ is a direct consequence. The same conclusion has been reached independently by Roy[3] using Vierbein geometry and intrinsic spin in general relativity. Thus, primeval charge e causes spacetime to spin, and generates both orbital and intrinsic angular momentum in spacetime itself. In the analysis of Roy,[3] the spin is Dirac's intrinsic spin, worked into general relativity.

The metric tensor $g_{\mu\nu}$ of Einstein's theory is related to the gravitational field tensor itself, and in the geometrical theory of electromagnetism in vacuo, the antisymmetric metric tensor $g_{\mu\nu}^{(A)}$, or, equivalently, the axial metric four-vector g_μ, is the electromagnetic field itself within a proportionality constant. In both cases the metric defines the curvature and so defines the field. If there is no curvature there is no field, which is another way of saying that if $\mathbf{B}^{(3)}$ is discarded, electrodynamics in vacuo cannot be described with general relativity. The metric in the special relativistic theory of vacuum electromagnetism is that of special relativity, which is diagonal. The novel metric g is purely off-diagonal, vividly demonstrating the fundamental conceptual difference between the general and special relativistic theories of vacuum electromagnetism. This is also the basic reason why $\mathbf{B}^{(3)}$ is a paradox in the Maxwellian point of view, which asserts simultaneously that $\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}$ exists but that $iB^{(0)}\mathbf{B}^{(3)*}$, to which it is equal, does not exist. In order to remove this paradox it is necessary to progress to general relativity and to use a geometrical basis for the theory of vacuum electromagnetism. The experimental observable $\mathbf{B}^{(3)}$ needs for its self-consistent explanation the use of covariant derivatives, and therefore spinning spacetime. Covariant derivatives are implicit in the structure of $G_{\mu\nu}$, which is proportional to the antisymmetric component of the curvature tensor itself. In this way, electrodynamics and gravitation become different geometrical aspects of curvature. The central importance of $\mathbf{B}^{(3)}$ in this deduction is seen through Eq. (25), in which the metric g_μ is directly proportional to B_μ, i.e., to $\mathbf{B}^{(3)}$ expressed in spacetime.

The above argument can be reversed in order to show that the existence of curved spacetime implies the existence of B_μ or $\mathbf{B}^{(3)}$. Fundamental tensorial arguments show[20] that there are symmetric and antisymmetric components of $R^\kappa_{\lambda\mu\nu}$. Both the gravitational and electromagnetic fields are determined in the geometrical theory by a change in the metric of spacetime, i.e., by a curvature of spacetime, and in the presence of electromagnetism and gravitation, spacetime cannot be galilean. A mass point in the gravitational field moves according to the symmetric part of $R^\kappa_{\lambda\mu\nu}$, obtained by $\kappa = \mu$. This symmetric part appears in the Einstein equation.[20] A charge point moves in the electromagnetic field according to the antisymmetric part of $R^\kappa_{\lambda\mu\nu}$, obtained by $\kappa = \lambda$, and this leads to Eq. (5), the minimal prescription in general relativity for vacuum electromagnetism. The equivalence principle in gravitation asserts that the gravitational field can be transformed away to zero in a frame in which covariant derivatives are replaced by ordinary partial derivatives, i.e., in which curvature vanishes, and in which curvilinear coordinates become locally Cartesian and Euclidean. Such a result in our geometrical theory of vacuum electromagnetism would imply the disappearance not only of $\mathbf{B}^{(3)}$ but of the whole of $G_{\mu\nu}$, meaning that there is no field component present. Our theory can therefore be *linearized*, by arbitrarily neglecting the presence of $\mathbf{B}^{(3)}$,

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but cannot reduce to Maxwellian electrodynamics because the latter is self-inconsistent.

Maxwellian electrodynamics in vacuo cannot be described by curved spacetime, but in the new geometrical theory there exists an equivalence principle between curvilinear coordinates and electromagnetic field, a principle which in its simplest and clearest form manifests itself through the fact that a charge spiralling in free space[3] with respect to a static reference frame is equivalent to a reference frame spiralling with respect to a static charge quantum e. Both points of view are equivalent. In the first, the test charge moves in an electromagnetic field, in the second, spacetime itself spins with respect to the charge, whose real spatial motion is neither uniform nor rectilinear, and whose geodesic (minimum path) is a spiral. The motion of the point charge is guided entirely, furthermore, by the field $\mathbf{B}^{(3)}$, which does not exist in Maxwellian theory. This again shows the fundamental difference of viewpoint between the two theories.

The connection between the metric tensor, affine parameter, and field tensor can be made through the single equation (5), so that electromagnetism becomes a property of spacetime determined by $g_{\mu\nu}^{(A)}$. By definition of the Riemann tensor,[20, 21]

$$g_{\mu\nu}^{(A)} = \frac{g}{R} R^{\lambda}_{\lambda\mu\nu} = \frac{g}{R} (\partial_{\mu} \Gamma^{\lambda}_{\lambda\nu} - \partial_{\nu} \Gamma^{\lambda}_{\lambda\mu} + \Gamma^{\rho}_{\lambda\mu} \Gamma^{\lambda}_{\rho\nu} - \Gamma^{\rho}_{\lambda\nu} \Gamma^{\lambda}_{\rho\mu}) \tag{26}$$

showing that if the affine parameters were zero, $g_{\mu\nu}^{(A)}$ itself would vanish. Therefore the electromagnetic field vanishes with the curvature of spacetime in precise analogy with the theory of gravitation. Equation (5), however, asserts that the metric tensor $g_{\mu\nu}^{(A)}$ is proportional to the field strength tensor defined by[2]

$$G_{\mu\nu} := \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} + \frac{e}{\hbar} (A_{\mu} A_{\nu}^{*} - A_{\nu} A_{\mu}^{*}) \tag{27}$$

where A_{μ} is the complex vector potential in vacuo.[1–3] The second term in Eq. (27) owes its existence to the use of covariant derivatives[2] and therefore vanishes if spacetime is not curved. We then recover, albeit inconsistently, the Maxwellian field strength tensor,

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \tag{28}$$

Comparison of Eqs. (26) and (27), using Eq. (5), leads to

$$\Gamma^{\lambda}_{\lambda\mu} = \Gamma^{\lambda}_{\mu\lambda} = \frac{e}{\hbar} A_{\mu} = \kappa_{\mu} \tag{29}$$

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which is the covariant form of the free-space minimal prescription, where κ_μ is the wave four-vector.[1-3] Equation (29) shows clearly that $\kappa_\mu = 0$ if the affine parameter is zero. The same comparison shows that

$$\Gamma^{\rho}_{\lambda\mu}\Gamma^{\lambda}_{\rho\nu} - \Gamma^{\rho}_{\lambda\nu}\Gamma^{\lambda}_{\rho\mu} = \left(\frac{e}{\hbar}\right)^2 (A_\mu A^*_\nu - A_\nu A^*_\mu) \tag{30}$$

so we obtain

$$g^{(A)}_{\mu\nu} = \frac{e}{\hbar}\frac{g}{R}\left(\partial_\nu A_\mu - \partial_\mu A_\nu + \frac{e}{\hbar}(A_\mu A^*_\nu - A_\nu A^*_\mu)\right) \tag{31}$$

which shows that the metric $g^{(A)}_{\mu\nu}$ is the electromagnetic field within a factor $eg/(\hbar R)$, as in the theory of gravitation. Indeed, g/R is *obtained* from the theory of gravitation itself because g/R is a property of curved spacetime.

There are several fundamental differences between Eq. (31) and the standard Maxwellian theory.

(1) Equation (31) is implicitly an equation of curved spacetime, depending directly on nonzero affine parameters. Even the *Maxwellian* part of Eq. (31), i.e., Eq. (28), depends on nonzero affine parameters through Eq. (29).

(2) Equation (26), the definition of Riemann's tensor, shows that $\mathbf{B}^{(3)}$ in the geometrical theory is nonzero in vacuo, and is given by the second term on the right-hand side of Eq. (31). Therefore, if Eq. (31) defines the electromagnetic field, $\mathbf{B}^{(3)}$ must be nonzero geometrically.

(3) If spacetime is curved in general, as accepted in the theory of gravitation, then Eq. (31) shows that Maxwell's theory, in which there occur no affine parameters, violates the geometry of curved spacetime.

(4) If charge is always associated in matter with mass, as seems to be the case, then spacetime must always ~~gibe~~ curved according to general relativity. However, according to Maxwellian theory, spacetime is always flat, even in the vicinity of matter, and so there is a fundamental incompatibility between the two theories. Equation (31), however, is fully compatible with curved spacetime, and does not, and should not, reduce to a Maxwellian point of view. This explains the paradox exposed by $\mathbf{B}^{(3)}$ in special relativity. be

(5) It is possible to *linearize* the geometrical theory of electromagnetism in vacuo by neglecting the $\mathbf{B}^{(3)}$ term. This gives the $F_{\mu\nu}$ tensor, which can be used as in standard electrodynamics. However, we recover from Eq. (31)

$$g^{(A)}_{\mu\nu} \to \frac{e}{\hbar}\frac{g}{R} F^{\mu}{}_{\nu} \tag{32}$$

$F_{\mu\nu}$

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a result which has no counterpart in Maxwellian electrodynamics because it is derived in curved spacetime. It is essential to understand that the linearized version of our theory gives all the results of standard electrodynamics through the standard field strength tensor $F_{\mu\nu}$, but is conceptually fundamentally different.

The equation of geodesic deviations[20] in general relativity may be written as

$$\frac{D^2\xi^\alpha}{d\tau} + R^\alpha_{\beta\gamma\delta} \frac{dx^\beta}{d\tau} \xi^\gamma \frac{dx^\delta}{d\tau} = 0 \tag{33}$$

where ξ^γ is a distance four-vector defined by[20]

$$\xi^\alpha := x^\alpha(B) - x^\alpha(A) \tag{34}$$

and τ is the proper time. Equation (33) is a fundamental description of curved spacetime in vacuo. By using the contraction $\alpha = \beta$, and using $R^\alpha_{\alpha\gamma\delta} = eG_{\gamma\delta}/\hbar$, Eq. (33) becomes

$$\frac{D^2\xi^\alpha}{d\tau^2} + \frac{e}{\hbar} G_{\gamma\delta} \frac{dx^\alpha}{d\tau} \xi^\gamma \frac{dx^\delta}{d\tau} = 0 \tag{35}$$

This is a *vacuum* equation which contains the charge e as part of the universal proportionality constant $e/\hbar$, and is similar in structure to the Lorentz force equation in conventional electrodynamics

$$\frac{D^2x^\alpha}{d\tau^2} - \frac{e}{m} F^\alpha_\beta \frac{dx^\alpha}{d\tau} = 0 \tag{36}$$

Equation (36) is for a charge in matter carried by a particle of mass m. Therefore the equivalent of the *Lorentz force equation* in vacuo may be written as

$$\left(R^\alpha_{\beta\gamma\delta} \frac{dx^\beta}{d\tau}\right) \xi^\gamma \frac{dx^\delta}{d\tau} = \left(\frac{e}{\hbar} G_{\gamma\delta} \frac{dx^\alpha}{d\tau}\right) \xi^\gamma \frac{dx^\delta}{d\tau} \tag{37}$$

which reduces to

$$R^\alpha_{\beta\gamma\delta} \frac{dx^\beta}{d\tau} = \frac{e}{\hbar} G_{\gamma\delta} \frac{dx^\alpha}{d\tau} \tag{38}$$

If $\alpha = \beta$ in Eq. (38) we recover, as we should,

$$R^{(A)}_{\gamma\delta} = \frac{e}{\hbar} G_{\gamma\delta} \tag{39}$$

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which is Eq. (5), a generalized version of the minimal prescription. In Eq. (39) there is no concept of acceleration being defined for a *particle*, because it is a vacuum equation and mass, m, does not appear in it. Electromagnetism is identified as the Riemann curvature tensor $R^{\alpha}_{\alpha\gamma\delta}$, and charge e need not be associated with a particle. This is consistent with the fact that the electromagnetic *field* itself is $\hat{C}$ negative.

If four-velocity is now defined as

$$v^{\beta} := \frac{dx^{\beta}}{d\tau} \tag{40}$$

then Eq. (38) becomes

$$R^{\alpha}_{\beta\gamma\delta} v^{\beta} = \frac{e}{\hbar} G_{\gamma\delta} v^{\alpha} \tag{41}$$

which again is a generalization of the minimal prescription applied to the free-field photon

$$\hbar\kappa = eA^{(0)} \tag{42}$$

These considerations reveal that the Riemann tensor $R^{\alpha}_{\beta\gamma\delta}$ exists in the complete absence of coordinates and so does the electromagnetic tensor $G_{\gamma\delta}$. In the conventional picture the $F_{\gamma\delta}$ tensor depends on the choice of coordinates.

In Eq. (41), if $\gamma = \delta$, then

$$R^{\alpha}_{\beta\gamma\gamma} v^{\beta} = 0 \tag{43}$$

and this is always true because $R^{\alpha}_{\beta\gamma\delta} = -R^{\alpha}_{\beta\delta\gamma}$. In Eq. (41), if $\alpha = \gamma$,

$$R^{\alpha}_{\beta\alpha\delta} v^{\beta} = \frac{e}{\hbar} G_{\alpha\delta} v^{\alpha} \tag{44}$$

and we obtain

$$R^{(S)}_{\beta\delta} v^{\beta} = R^{(A)}_{\alpha\delta} v^{\alpha} \tag{45}$$

Equation (45) is a very clear form of the Lorentz force equation, relating the symmetric and antisymmetric Ricci tensors through the velocity v^{σ}. It is the equation linking the symmetric and antisymmetric parts of the same Ricci tensor and therefore relating the gravitational and electromagnetic fields.

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The equation of motion for vacuum electromagnetism in general relativity can be written in a physically transparent form by starting from Eq. (11b) and using

$$R^{(A)}_{\mu\nu} = \frac{e}{\hbar} G_{\mu\nu} = \frac{\kappa^2}{\hbar} J_{\mu\nu} \tag{46}$$

We immediately obtain the physically transparent result,

$$D_\nu J^{\mu\nu} = 0 \tag{47}$$

which when written out in terms of the affine parameter becomes

$$\frac{\partial J^{\mu\nu}}{\partial x^\nu} + \Gamma^\mu_{\sigma\nu} J^{\sigma\nu} + \Gamma^\nu_{\sigma\nu} J^{\mu\sigma} = 0 \tag{48}$$

or

$$j^\lambda_{;\mu} := D_\mu J^\lambda = \frac{\partial J^\lambda}{\partial x^\mu} + \Gamma^\lambda_{\mu\nu} J^\nu = 0 \tag{49}$$

and is the same form precisely as the equation of linear four-velocity in general relativity, e.g., Ref. 21, Eq. (87.2). Therefore electromagnetism in vacuo in general relativity is an equation of motion for angular momentum in curvilinear coordinates, i.e., an equation of spin. The angular momentum is a property of spinning spacetime itself through Eq. (46). Finally, dividing Eq. (49) by the line element *ds* of general relativity, we obtain

$$\frac{\partial J^\lambda}{\partial s} = -\Gamma^\lambda_{\mu\nu} J^\nu \frac{\partial x^\mu}{\partial s} \tag{50}$$

which is an equation in torque equivalent to a description of the angular four-acceleration of charge.

4. CONCLUDING REMARKS

Equation (29), $\kappa_\mu = \Gamma^\lambda_{\lambda\mu}$, contains the essence of the geometrical theory. The affine parameter $\Gamma^\lambda_{\lambda\mu}$ of curved spacetime is identified as the wavenumber κ_μ of vacuum electromagnetic radiation. The affine parameter vanishes however, in a frame of reference in which curvature is zero. From Eq. (29), this frame corresponds to $\kappa_\mu = 0$, in which case there is no electromagnetic radiation, and in which case $A_\mu = 0$. Therefore our geometric theory of vacuum electromagnetism identifies it with spacetime curvature.

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If curvature is zero, $R_{\mu\nu}^{(A)}$ is zero, and so is $G_{\mu\nu}$. The present theory is distinguished from Maxwellian electrodynamics by the self-consistent consideration given to the $\mathbf{B}^{(3)}$ field. In order to progress from the Maxwellian theory (special relativity) to the geometrical theory (general relativity) it must be accepted that $\mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = iB^{(0)}\mathbf{B}^{(3)*}$, and that $\mathbf{B}^{(3)}$ is an experimental observable under the right conditions.[1-12] If so, gravitation becomes unified with electromagnetism.

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