

The resonant Coulomb and Ampère-Maxwell law

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Abstract

Many experiments and theoretical considerations have emerged with the aim of extracting energy from spacetime, aether or vacuum. The working mechanism is not explainable by standard physics principles. In this paper, we present mechanisms of Einstein-Cartan-Evans (ECE) theory that are able to explain this energy extraction on a level of general relativity. In ECE theory, spacetime curvature and torsion appear in the fundamental equations of electromagnetism. When these equations are written in terms of potentials, resonances appear in their solutions that are not present in standard electrodynamics. These resonances suggest possible ways of energy transfer from spacetime, especially resonances in the generalized Coulomb and Ampère-Maxwell laws.

Keywords: ECE theory, energy from spacetime, Maxwell's equations, Coulomb law, Ampère-Maxwell law.

1 Introduction

In the search for new energy sources, the idea of producing energy through resonance effects has often been discussed. One candidate for the energy source is the physical vacuum, which is non-empty. The energy density of the vacuum is indeed very high, as has been found in experiments with elementary particles. In ECE theory [1-3], the non-empty vacuum is identified with curved and twisted spacetime on the classical level. The internal structure of spacetime appears as potentials without macroscopic force fields. This internal structure may also be interpreted as a kind of aether. In this way it becomes plausible that a lot of energy is there in the form of vacuum fluctuations or a hydrodynamic background flux [4].

The idea of gaining energy from spacetime has already been pursued in many papers of ECE theory, starting with [5]. It has been shown that the homogeneous

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current (a kind of magnetic current) can only appear in a spacetime with a non-vanishing spin connection. Resonances were investigated using the Coulomb law (primarily) [6–10], often by numerical methods. This law was also used in a popular paper on the electromagnetic sector of the Evans field theory [11].

In the current paper, we develop analytical methods to describe resonances, first in the Coulomb law and then in the Ampère-Maxwell law. The latter was not investigated before, in this respect, except in [5]. With the aid of computer algebra, we are able to present analytical solutions and graphics that were not reported before. Both of these equations of electrodynamics, when written in terms of the potential, can be reduced to a form of Euler-Bernoulli resonance equations.

2 Spin connection resonance in the Coulomb law

We consider the resonant Coulomb law. One of the most important consequences of general relativity applied to electrodynamics is that the spin connection enters the relation between the field and potential. In classical electrodynamics, this relation reads

$$\mathbf{E} = -\nabla\phi - \frac{\partial\mathbf{A}}{\partial t}, \quad (1)$$

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad (2)$$

where \mathbf{E} is the electric field, \mathbf{B} the magnetic induction, \mathbf{A} the vector potential and ϕ the electric scalar potential. In ECE theory, the field quantities have a polarization index a and are augmented by expressions of the spin connections. These are, in vector notation, the vector spin connection ω^a_b and the scalar spin connection ω^a_{0b} . The above relations then read:

$$\mathbf{E}^a = -\nabla\phi^a - \frac{\partial\mathbf{A}^a}{\partial t} - c\omega^a_{0b}\mathbf{A}^b + \omega^a_b\phi^b, \quad (3)$$

$$\mathbf{B}^a = \nabla \times \mathbf{A}^a - \omega^a_b \times \mathbf{A}^b. \quad (4)$$

The classical Coulomb law is

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}. \quad (5)$$

Assuming the absence of a vector potential (absence of a magnetic field), the electric field in the standard model is

$$\mathbf{E} = -\nabla\phi, \quad (6)$$

where ϕ is the electric potential. Under the same assumption, the electric field in ECE theory, according to Eq. (3), is

$$\mathbf{E} = -\nabla\phi + \omega\phi, \quad (7)$$

where we have written the fields without polarization index. The vector spin connection ω then has identical indices which can be omitted. Therefore, Eq. (5) takes the form

$$\nabla^2\phi - \omega \cdot \nabla\phi - (\nabla \cdot \omega)\phi = -\frac{\rho}{\epsilon_0}. \quad (8)$$

The equivalent equation in the standard model is the Poisson equation, which is a limit of Eq. (8) when the spin connection is zero. The Poisson equation does not give resonant solutions. However, Eq. (8) has resonant solutions of Euler-Bernoulli type, as can be seen in the following discussion. Restricting consideration to one cartesian coordinate, we have only the dependencies $\phi(X)$ and the spin connection has only an X component $\omega_X(X)$. Then Eq. (8) reads:

$$\frac{d^2\phi}{dX^2} - \omega_X \frac{d\phi}{dX} - \frac{d\omega_X}{dX} \phi = -\frac{\rho}{\epsilon_0}. \quad (9)$$

This equation has the structure of a damped Euler-Bernoulli resonance of the form

$$\frac{d^2\phi}{dx^2} + \alpha \frac{d\phi}{dx} + \kappa_0^2 \phi = F_0 \cos(\kappa x), \quad (10)$$

if we assume $\omega_X < 0$. Below we will see that this is not a real restriction. κ_0 is the spatial eigenfrequency, measured in 1/m, and α is the damping constant. At the right-hand side, there is a periodic driving force with spatial frequency (wave number) κ . The particular solution of this differential equation is

$$\phi = F_0 \frac{\alpha \kappa \sin(\kappa x) + (\kappa_0^2 - \kappa^2) \cos(\kappa x)}{(\kappa_0^2 - \kappa^2)^2 + \alpha^2 \kappa^2}. \quad (11)$$

For vanishing damping, we have

$$\phi \rightarrow F_0 \frac{\cos(\kappa x)}{(\kappa_0^2 - \kappa^2)^2}. \quad (12)$$

For $\kappa \rightarrow \kappa_0$ the amplitude of $\phi(x)$ approaches infinity. In the case of damping, the amplitude in the resonance point remains finite (see examples in Fig. 1).

By comparing Eqs. (9) and (10), it is seen that the Coulomb equation (8) has no constant coefficients and thus is not an original form of the Euler-Bernoulli resonance. Therefore, we can expect that the solutions may differ significantly from those of the original Euler-Bernoulli equation. To investigate this, we consider an example in spherical polar coordinates. We assume that the potential and the spin connection depend only on the radial coordinate r . For the radial (and only) component of the spin connection we assume

$$\omega_r = \frac{1}{r}. \quad (13)$$

The differential operators in (8) then take the form

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r} \frac{\partial \phi}{\partial r}, \quad (14)$$

$$\nabla \phi = \frac{\partial \phi}{\partial r} \cdot \mathbf{e}_r, \quad (15)$$

$$\nabla \cdot (\omega_r \mathbf{e}_r) = -\frac{1}{r^2}. \quad (16)$$

Then, Eq. (8), with the right-hand side replaced by an oscillatory driving term, reads:

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} - \frac{\phi}{r^2} = F_0 \sin(\kappa r). \quad (17)$$

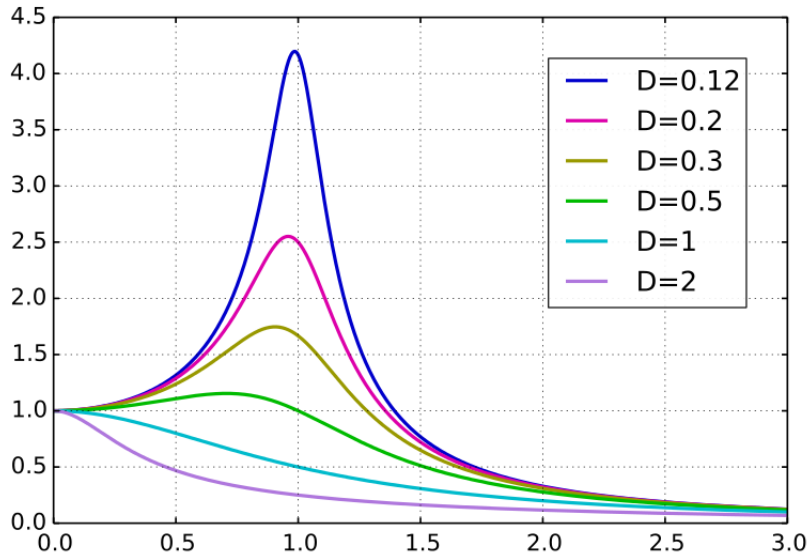


Figure 1: Y axis: steady-state amplitude $\phi/\phi_{\text{static}}$ of a damped driven oscillator with different damping constants $D = \alpha/2$. X axis: frequency ratio κ/κ_0 [14].

This equation can be solved analytically. (see computer algebra code [15]). The solution contains an expression $-\cos(\kappa r)/r$, leading to the limit $\phi(r) \rightarrow -\infty$ in the case where $r \rightarrow 0$ as graphed in Fig. 2. Other model examples for ω are listed in the code. Although the Coulomb law with the spin connection term resembles a resonance equation with damping, there is no damping for $r \rightarrow 0$ because of the non-constant coefficients in the equation. Computer algebra shows that the sign of ω_r does not significantly change the type of solutions, although this would be the case for the Euler-Bernoulli equation (10).

The spin connection has already been incorporated during the course of development of ECE theory into the Coulomb law, which is the basic law used in the development of quantum chemistry. This process has been illustrated with the hydrogen atom [7]. It serves as a model system for the huge class of atomic, molecular and solid-state physics. (The most used method for computation of electronic properties of solids is Density Functional Theory.)

The ECE theory has also been used to design or explain circuits, which use spin connection resonance to take power from spacetime, notably in papers 63 and 94 of the ECE series on www.aias.us [7,10]. In paper 63, the spin connection was incorporated into the Coulomb law and the resulting equation in the scalar potential shown to have resonance solutions using an Euler transform method. In paper 94, this method was extended and applied systematically to the Bedini machine, which was shown to have the chance of producing energy from spacetime, although nobody has succeeded in achieving this to date. In addition, spacetime effects in transformers have been found by Ide and successfully explained by ECE theory [12].

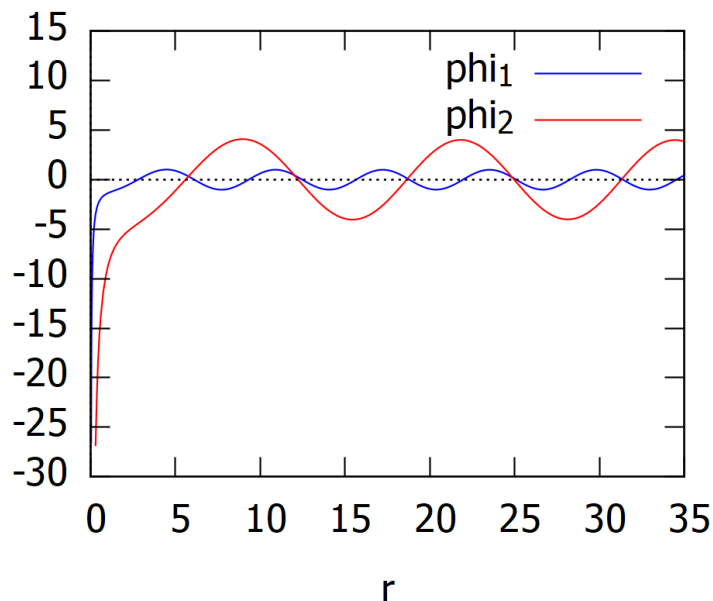


Figure 2: Solution of Eq. (17), $\kappa = 1$ and $\kappa = 0.5$, other constants normalized.

3 Spin connection resonance in the Ampère-Maxwell law

As another important example, we consider resonant forms of the Ampère-Maxwell law. In potential representation [3], this law reads

$$\begin{aligned} & \nabla (\nabla \cdot \mathbf{A}^a) - \nabla^2 \mathbf{A}^a - \nabla \times (\boldsymbol{\omega}^a_b \times \mathbf{A}^b) \\ & + \frac{1}{c^2} \left(\frac{\partial^2 \mathbf{A}^a}{\partial t^2} + c \frac{\partial (\boldsymbol{\omega}^a_{0b} \mathbf{A}^b)}{\partial t} + \nabla \frac{\partial \phi^a}{\partial t} - \frac{\partial (\boldsymbol{\omega}^a_b \phi^b)}{\partial t} \right) = \mu_0 \mathbf{J}^a. \end{aligned} \quad (18)$$

\mathbf{J}^a is a current, which may have a polarization dependence. Assuming the simple case that there is no scalar potential, and that the vector potential is independent of space location and has a pure time dependence only, we obtain the equation

$$\frac{\partial^2 \mathbf{A}^a}{\partial t^2} + c \frac{\partial (\boldsymbol{\omega}^a_{0b} \mathbf{A}^b)}{\partial t} = \frac{1}{\epsilon_0} \mathbf{J}^a. \quad (19)$$

Restricting this equation to one polarization index, we have

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} + c \frac{\partial (\omega_0 \mathbf{A})}{\partial t} = \frac{1}{\epsilon_0} \mathbf{J}. \quad (20)$$

This equation is formally identical to (9), but with correct signs as in (10), except that it is a vector equation and the (only) coordinate is the time coordinate. Here, the spin connection is the scalar spin connection ω_0 with units of 1/m. We replace it by a time frequency, subsuming the factor c :

$$\omega_t = c \omega_0 \quad (21)$$

so that (20) can be written:

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} + \omega_t \frac{\partial \mathbf{A}}{\partial t} + \frac{\partial \omega_t}{\partial t} \mathbf{A} = \frac{1}{\epsilon_0} \mathbf{J}, \quad (22)$$

in full analogy to Eq. (9/10). Therefore, the existence of the time spin connection makes the Ampère-Maxwell law a resonance equation in the same way as discussed for the Coulomb law in the preceding section.

Another resonance is possible, when we assume that the vector potential is only space-dependent and there is no scalar potential, for example in magnetic structures. If \mathbf{A} is divergence-free, we obtain from Eq. (18), again for one direction of polarization:

$$-\nabla^2 \mathbf{A} - \nabla \times (\boldsymbol{\omega} \times \mathbf{A}) = \mu_0 \mathbf{J}. \quad (23)$$

Here the vector spin connection $\boldsymbol{\omega}$ appears again. Using the vector identity

$$\nabla \times (\mathbf{a} \times \mathbf{b}) = \mathbf{a}(\nabla \cdot \mathbf{b}) - \mathbf{b}(\nabla \cdot \mathbf{a}) + (\mathbf{b} \cdot \nabla) \mathbf{a} - (\mathbf{a} \cdot \nabla) \mathbf{b} \quad (24)$$

and that \mathbf{A} is divergence-free, we obtain

$$\nabla \times (\boldsymbol{\omega} \times \mathbf{A}) = -\mathbf{A}(\nabla \cdot \boldsymbol{\omega}) + (\mathbf{A} \cdot \nabla) \boldsymbol{\omega} - (\boldsymbol{\omega} \cdot \nabla) \mathbf{A} \quad (25)$$

so that Eq. (23) becomes

$$\nabla^2 \mathbf{A} + \mathbf{A}(\nabla \cdot \boldsymbol{\omega}) - (\mathbf{A} \cdot \nabla) \boldsymbol{\omega} + (\boldsymbol{\omega} \cdot \nabla) \mathbf{A} = -\mu_0 \mathbf{J}. \quad (26)$$

It can be seen that this equation contains differentiations of \mathbf{A} in zeroth, first and second order. Obviously, resonances are possible for this special form of the Ampère-Maxwell law. To show this, we first define a special system, where \mathbf{A} is restricted to two dimensions and the spin connection is perpendicular to the plane of \mathbf{A} . In cartesian coordinates, we then have

$$\mathbf{A} = \begin{bmatrix} A_X \\ A_Y \\ 0 \end{bmatrix}, \quad \boldsymbol{\omega} = \begin{bmatrix} 0 \\ 0 \\ \omega_Z \end{bmatrix}, \quad \mathbf{J} = \begin{bmatrix} J_X \\ J_Y \\ J_Z \end{bmatrix}, \quad (27)$$

where all variables depend on coordinates X and Y . As shown in computer algebra code [15], it follows that

$$\nabla^2 \mathbf{A} = \begin{bmatrix} \frac{\partial^2 A_X}{\partial X^2} + \frac{\partial^2 A_X}{\partial Y^2} \\ \frac{\partial^2 A_Y}{\partial X^2} + \frac{\partial^2 A_Y}{\partial Y^2} \\ 0 \end{bmatrix}, \quad (28)$$

$$\boldsymbol{\omega} \times \mathbf{A} = \begin{bmatrix} -A_Y \omega_Z \\ A_X \omega_Z \\ 0 \end{bmatrix}, \quad (29)$$

$$\nabla \times (\boldsymbol{\omega} \times \mathbf{A}) = \begin{bmatrix} 0 \\ 0 \\ \frac{\partial \omega_Z}{\partial X} A_X + \frac{\partial \omega_Z}{\partial Y} A_Y + \frac{\partial A_X}{\partial X} \omega_Z + \frac{\partial A_Y}{\partial Y} \omega_Z \end{bmatrix}. \quad (30)$$

Inserting this into Eq. (23) leads to three component equations:

$$\frac{\partial^2 A_X}{\partial X^2} + \frac{\partial^2 A_X}{\partial Y^2} = -\mu_0 J_X, \quad (31)$$

$$\frac{\partial^2 A_Y}{\partial X^2} + \frac{\partial^2 A_Y}{\partial Y^2} = -\mu_0 J_Y, \quad (32)$$

$$\frac{\partial \omega_Z}{\partial X} A_X + \frac{\partial \omega_Z}{\partial Y} A_Y + \frac{\partial A_X}{\partial X} \omega_Z + \frac{\partial A_Y}{\partial Y} \omega_Z = -\mu_0 J_Z. \quad (33)$$

The first two equations decouple from the third, which is of first order in derivatives only. In order to get an impression of how resonances can occur, we simplify this equation set further, so that only one variable $A_X(X)$ is left:

$$\mathbf{A} = \begin{bmatrix} A_X \\ 0 \\ 0 \end{bmatrix}, \quad (34)$$

where $\boldsymbol{\omega}$ and \mathbf{J} remain as in (27) but depend on the X variable only. Then the equation set (31-33) simplifies to

$$\frac{\partial^2 A_X}{\partial X^2} = -\mu_0 J_X, \quad (35)$$

$$0 = -\mu_0 J_Y, \quad (36)$$

$$\frac{\partial \omega_Z}{\partial X} A_X + \frac{\partial A_X}{\partial X} \omega_Z = -\mu_0 J_Z. \quad (37)$$

From Eq. (36) follows $J_Y = 0$ as a constraint. Eqs. (35) and (37) are not compatible any more, but we add both equations to obtain an analytically solvable equation that combines the properties of both equations:

$$\frac{\partial^2 A_X}{\partial X^2} + \frac{\partial A_X}{\partial X} \omega_Z + \frac{\partial \omega_Z}{\partial X} A_X = -\mu_0 (J_X + J_Z). \quad (38)$$

(Below, we will see that this is a meaningful operation.) This is a resonance equation with non-constant coefficients, as were Eqs. (9) and (22). For demonstration, we present some solutions for this equation in cartesian coordinates. In Table 1 we show four solutions of Eq. (38) for given combinations of current density \mathbf{J} and spin connection ω_Z . These are graphed in Fig. 3. All solutions have divergence points for $X \rightarrow 0$, $X \rightarrow \pm\infty$, or elsewhere. Eq. (35) has an oscillatory solution as expected but Eq. (37), although only of first order, has diverging solutions (solutions 5-7, see Table 1 and Fig. 4). Therefore, the combined equation (38) is an approximation to a full-blown calculation where all components of \mathbf{A} and $\boldsymbol{\omega}$ are present, as in Eq. (23).

It is known from the work of Tesla, for example, that strong resonances in electric power can be obtained with a suitable apparatus, and such resonances cannot be explained using the standard model. One consistent explanation of Tesla's well-known results is given by the incorporation of the spin connection into classical electrodynamics.

Equation	Fig. ref.	J_X, J_Z	ω_Z	Solution
(38)	solution 1	J_0	$1/X$	$\frac{2}{3}J_0\mu_0X^2$
	solution 2	J_0/X	$1/X$	$-J_0\mu_0(X \log(X) + \frac{1}{2}X)$
	solution 3	$J_0 \sin(aX)$	$1/X$	$2\frac{J_0\mu_0}{a^3}(\sin(aX) + \frac{\cos(aX)}{X})$
	solution 4	$J_0 X^2$	$1/X$	$\frac{2}{15}J_0\mu_0X^4$
(35)	solution 5	$J_0 \cos(\kappa_0 X)$		$\frac{\mu_0}{\kappa_0^2} \cos(\kappa_0 X)$
(37)	solution 6	$J_0 \cos(\kappa_0 X)$	$\kappa_0 \cos(\kappa X)$	$-X \frac{\mu_0}{\kappa_0^2} \frac{\sin(\kappa_0 X)}{\cos(\kappa X)}$
	solution 7	$J_0 \cos(\kappa_0 X)$	$1/X$	$-X \frac{\mu_0}{\kappa_0} \sin(\kappa_0 X)$

Table 1: Solutions of model resonance equations.

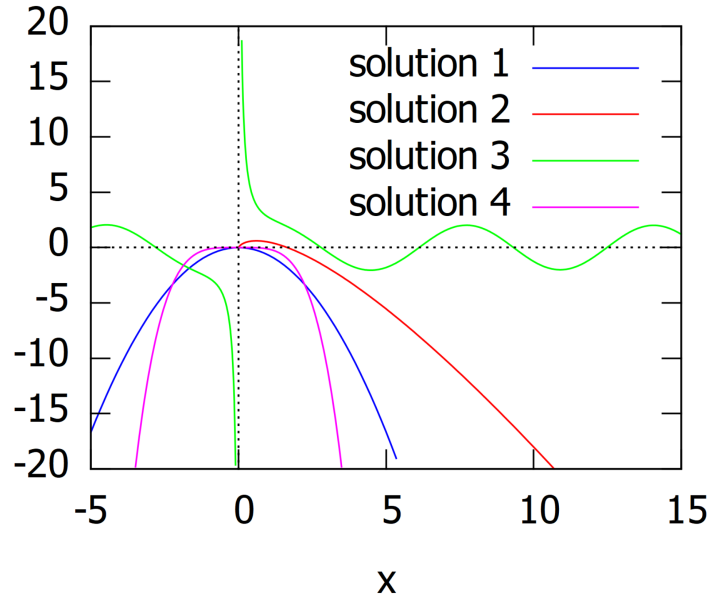


Figure 3: Solutions of Eq. (38), all constants normalized.

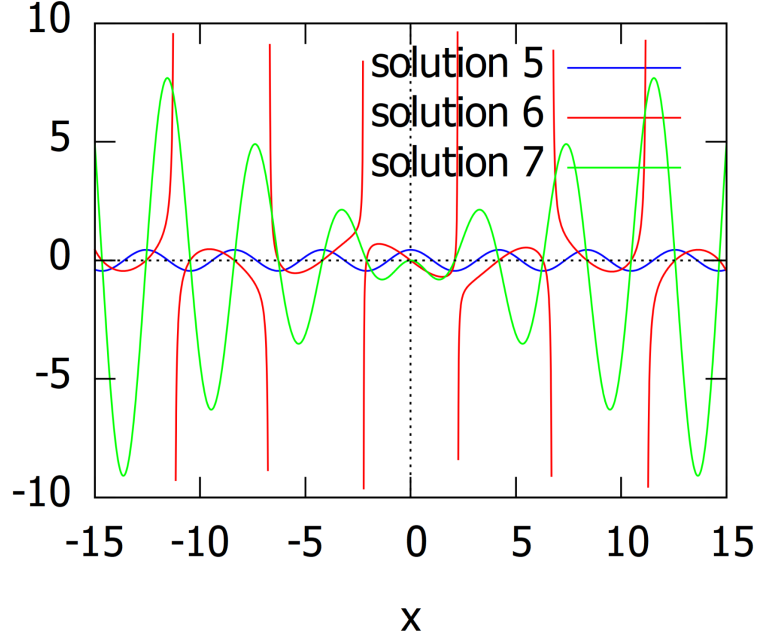


Figure 4: Solutions of Eqs. (35) and (37) with $\kappa_0 = 1.5$ and $\kappa = 0.7$, other constants normalized.

4 Additional approaches for extracting energy from spacetime

We have used resonance mechanisms to explain and predict effects unknown in standard electrodynamics. Sometimes it is argued that a current induced by unknown electrodynamical effects behaves different from a “standard” current. It is called a “cold current” because it allegedly does not obey thermodynamic laws and cools the apparatus, instead of producing thermal heating. If we assume that such a current exists, it has to be derived from non-standard terms of electrodynamics. ECE theory offers a possibility for explaining this aspect, which was investigated by D. W. Lindstrom in detail [13]. Here, however, we concentrate on the different aspect of how this is related to resonances.

Let’s separate the Ampère-Maxwell law of ECE theory (18) into standard terms (without spin connections) and ECE terms (with spin connections). Then we can write

$$\nabla(\nabla \cdot \mathbf{A}^a) - \nabla^2 \mathbf{A}^a + \frac{1}{c^2} \left(\frac{\partial^2 \mathbf{A}^a}{\partial t^2} + \nabla \frac{\partial \phi^a}{\partial t} \right) = \mu_0 \mathbf{J}_1^a, \quad (39)$$

$$-\nabla \times (\boldsymbol{\omega}^a_b \times \mathbf{A}^b) + \frac{1}{c^2} \left(c \frac{\partial (\boldsymbol{\omega}^a_{0b} \mathbf{A}^b)}{\partial t} - \frac{\partial (\boldsymbol{\omega}^a_b \phi^b)}{\partial t} \right) = \mu_0 \mathbf{J}_2^a \quad (40)$$

with

$$\mathbf{J}^a = \mathbf{J}_1^a + \mathbf{J}_2^a. \quad (41)$$

The sum of both equations (39) and (40) gives the original Ampère-Maxwell law of ECE theory (18). Eq. (39) is the classical Ampère-Maxwell law with \mathbf{J}_1^a , while Eq. (40) with \mathbf{J}_2^a could be interpreted as an equation for the “cold current”. This is an equation with first-order derivatives in space and time.

If we use the Lorenz gauge

$$\nabla \cdot \mathbf{A}^a + \frac{1}{c^2} \frac{\partial \phi^a}{\partial t} = 0, \quad (42)$$

Eq. (39) becomes the inhomogeneous wave equation, as is well known:

$$\frac{1}{c^2} \frac{\partial^2 \mathbf{A}^a}{\partial t^2} - \nabla^2 \mathbf{A}^a = \mu_0 \mathbf{J}_1^a. \quad (43)$$

For the “cold current equation” (40) we can assume that there is no scalar potential and no time dependence. Then we obtain

$$-\nabla \times (\boldsymbol{\omega} \times \mathbf{A}) = \mu_0 \mathbf{J}_2^a, \quad (44)$$

which is Eq. (23) without the term $-\nabla^2 \mathbf{A}^a$. This equation was already listed in [13]. For a solution, we can make the same simplification as for Eq. (34). Then we obtain Eq. (37) exactly:

$$\frac{\partial \omega_Z}{\partial X} A_X + \frac{\partial A_X}{\partial X} \omega_Z = -\mu_0 J_Z. \quad (45)$$

Two solutions of this equation are given in Table 1 and graphed in Fig. 4. Interestingly, this theory of a “cold current” delivers resonant solutions, as does the Ampère-Maxwell equation. Obviously, it seems possible that in certain situations, where a suitable spin connection has been experimentally provided, a novel type of resonance could be realized. However, it has to be considered that in this case the current would not prerequisite of the resonance but rather a consequence. The current would not be used here to define a resonance condition; instead it would be the consequence of a configuration of suitable \mathbf{A} and $\boldsymbol{\omega}$ fields. An open question is how a certain spin connection can be prepared experimentally. This subject has to be worked out further by theory and experiment.

That the current is “cold” does not mean that there is a violation of the laws of physics. Rather, it has to be assumed that the system under consideration is an open system, and the vacuum or spacetime has to be included to make it a closed system, i.e., achieve conservation of energy. Thus, there is no conflict with the laws of thermodynamics.

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