The homogeneous and inhomogeneous ECE current, Part I

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Abstract

ECE theory gives us a symmetric form of the Maxwell-like field equations with two types of currents: the inhomogeneous current, which is identical to the usual current of electric charge carriers, and the homogeneous current, which is not present in standard theory. In this paper, we present and discuss both currents on the basis of ECE2 theory. In addition to other terms, a conductivity term is also present in ECE theory, in contrast to standard electrodynamics. The full duality of the electric and magnetic field becomes evident. It is also shown that the homogeneous current is a source of electromagnetic wave propagation.

Keywords: ECE theory, ECE2 theory, electrodynamics, homogeneous and inhomogeneous current.

1 Introduction

At the present time, standard electrical theory is completely based on Maxwell's equations, which are compatible with Einstein's special relativity. The Lorentz force, which is not included in Maxwell's equations, can also be derived from special relativity. Maxwell's equations, notated in the vector form of Heaviside, are considered to be a complete and well-defined theoretical basis of electromagnetism. Potentials are included only as "auxiliary means for computation", although it is known from quantum mechanics that potentials are the cause of excitations in electronic systems, and are not "regaugable" as assumed by standard electromagnetic theory.

Einstein-Cartan-Evans (ECE) theory [1–3] has changed this picture considerably. The field equations of ECE theory are Maxwell-like, but valid in a spacetime of general relativity with curvature and torsion. They are symmetric in the sense that, besides the electric charge and current density, there is also a magnetic charge and current density, as Paul Dirac had inferred earlier. In ECE

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theory, the full duality of these equations becomes visible. The two currents are called the homogeneous and inhomogeneous current, for historical reasons. In standard theory, the homogeneous current (the right side of the Faraday law) was interpreted as a current of magnetic monopoles. We will show in this series of papers that this is an incomplete view. When the vacuum is included in the interpretation, we find polarization effects through this current. For example, this interpretation can explain the experimental work of Nicola Tesla.

2 Field equations

In the following discussion, we will compare the field equations of ECE theory to those of standard theory. As a hint about an upcoming interpretation, we mention you will see the Volt-second (or Weber) where you might expect to see a different unit (and that one Weber per square meter is one Tesla).

2.1 Differences between standard theory and ECE theory

We start with Maxwell's equations from standard theory:

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \qquad \text{Gauss law,} \tag{1}$$

$$\frac{\partial \mathbf{B}}{\partial t} + \boldsymbol{\nabla} \times \mathbf{E} = \mathbf{0} \qquad \text{Faraday law,} \tag{2}$$

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \text{Coulomb law,} \tag{3}$$

$$-\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \boldsymbol{\nabla} \times \mathbf{B} = \mu_0\,\mathbf{J} \qquad \text{Ampère-Maxwell law.} \tag{4}$$

$\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic induction, ρ is the electric charge density and $\mathbf{J}$ is the electric current density.

Maxwell's theory is a theory of special relativity; it has been shown to be Lorentz-covariant, i.e., the equations keep their form when the coordinate system underlying the fields undergoes a Lorentz transformation.

In ECE theory, the field equations are derived from Cartan geometry, and they can be transformed into vector notation that corresponds to Maxwell's equations. In their simplest form, these field equations read

$$\boldsymbol{\nabla} \cdot \mathbf{B} = -\mu_0 j^0, \tag{5}$$

$$\frac{\partial \mathbf{B}}{\partial t} + \boldsymbol{\nabla} \times \mathbf{E} = c\,\mu_0\,\mathbf{j}, \tag{6}$$

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}, \tag{7}$$

$$-\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \boldsymbol{\nabla} \times \mathbf{B} = \mu_0\,\mathbf{J}. \tag{8}$$

In addition to the components of standard Maxwellian theory, a charge density j^0 and current density $\mathbf{j}$ appear. The physical units in Maxwell's equations that we will be using are

$$[\boldsymbol{\nabla} \cdot \mathbf{B}] = \frac{T}{m} = \frac{Vs}{m^3}, \tag{9}$$

$$[\boldsymbol{\nabla} \cdot \mathbf{E}] = \frac{V}{m^2}, \tag{10}$$

$$[\mu_0] = \frac{Vs}{Am}, \tag{11}$$

from which we get the following:

$$[j^0] = \frac{T}{m} = \frac{A}{m^2}, \tag{12}$$

$$[\mathbf{j}] = \frac{A}{ms}. \tag{13}$$

The constant μ_0 has been introduced for convenience. Redefining both j^0 and $\mathbf{j}$ without this constant leads to units of

$$[\frac{j^0}{\mu_0}] = \frac{Vs}{m^3}, \tag{14}$$

$$[\frac{\mathbf{j}}{\mu_0}] = \frac{V}{m^2} = \frac{Vs}{m^2 s}. \tag{15}$$

In this case, it can be seen that j^0 is a volume density, while $\mathbf{j}$ is a flux density vector (flux per area). Compared to the usual electric charge density and current density units, C/m^3 and $C/(m^2 s)$, it is seen that the electric charge is replaced by the product of Volt times second, also called Weber. This suggests that we should consider j^0 and $\mathbf{j}$ as volume and current densities of "magnetic charges". With respect to units, the Coulomb is replaced by the Volt-second. Whether such charges exist is currently an unresolved question, but there are some hints that they can appear at the quantum level.

In standard theory, charge and current densities are external sources that are independent of the **E** and **B** fields. This is analogous to the mass density in Einstein's general relativity, which is the source of the gravitational field. In ECE theory, however, there are no external sources. All source terms are field quantities, and while their combinations may behave like local sources, their intrinsic nature remains that of a field quantity. This avoids problems that arise, for example, in Einstein's general relativity, where fields have their own energy density, and act as additional sources, so energy is not conserved. This problem is not present in ECE theory, because sources are fields, exclusively.

This is an essential difference between ECE theory and standard electrodynamics that is often not understood. ECE electrodynamics is a theory of general relativity, and the field equations, although formally identical to those of standard theory, are generally covariant instead of only Lorentz covariant.

Because there are no external sources in ECE theory, all density and current terms in Eqs. (5-8) can be expressed by fields [3]. The right sides of these equations then become

$$-\mu_0 j^0 = 2\left(\frac{1}{W^{(0)}}\mathbf{A} - \boldsymbol{\omega}_{(\Lambda)}\right)\cdot\mathbf{B}, \tag{16}$$

$$c\mu_0 \mathbf{j} = 2\left(\left(\frac{1}{W^{(0)}}\phi - c{\omega_{(\Lambda)}}^0\right)\mathbf{B} - \left(\frac{1}{W^{(0)}}\mathbf{A} - \boldsymbol{\omega}_{(\Lambda)}\right)\times\mathbf{E}\right), \tag{17}$$

$$\frac{\rho}{\epsilon_0} = -2\left(\frac{1}{W^{(0)}}\mathbf{A} - \boldsymbol{\omega}\right)\cdot\mathbf{E}, \tag{18}$$

$$\mu_0 \mathbf{J} = 2\left(\left(\frac{1}{c^2 W^{(0)}}\phi - \frac{1}{c}\omega^0\right)\mathbf{E} + \left(\frac{1}{W^{(0)}}\mathbf{A} - \boldsymbol{\omega}\right)\times\mathbf{B}\right). \tag{19}$$

We see that, in addition to the **E** and **B** fields, the vector potential **A** and scalar potential ϕ also appear. In ECE theory, the potentials are interpreted as spacetime (or "aether") flux and pressure. This is a first hint that material charges consist of localized aether structures. We also see quantities denoted by ω. These quantities are called spin connections and they describe spacetime curvature and torsion. ω^0 and ${\omega_{(\Lambda)}}^0$ are scalar spin connections, and $\boldsymbol{\omega}$ and $\boldsymbol{\omega}_{(\Lambda)}$ are vector spin connections. $W^{(0)}$ is a constant with units of Vs. The spin connections have units of inverse meters. Two spin connections have an index Λ and are different[1] from those without this index.

The field equations can be simplified and made more understandable by introducing wave numbers (in scalar and vector form) defined by

$$\kappa_{(\Lambda)0} = \frac{1}{cW^{(0)}}\phi - {\omega_{(\Lambda)}}^0, \tag{20}$$

$$\boldsymbol{\kappa}_{(\Lambda)} = \frac{1}{W^{(0)}}\mathbf{A} - \boldsymbol{\omega}_{(\Lambda)}, \tag{21}$$

$$\kappa_0 = \frac{1}{cW^{(0)}}\phi - \omega^0, \tag{22}$$

$$\boldsymbol{\kappa} = \frac{1}{W^{(0)}}\mathbf{A} - \boldsymbol{\omega}. \tag{23}$$

Then, Eqs. (5 - 8) can be written as

$$\boldsymbol{\nabla}\cdot\mathbf{B} = 2\boldsymbol{\kappa}_{(\Lambda)}\cdot\mathbf{B}, \tag{24}$$

$$\frac{\partial\mathbf{B}}{\partial t} + \boldsymbol{\nabla}\times\mathbf{E} = 2\left(c\kappa_{(\Lambda)0}\mathbf{B} - \boldsymbol{\kappa}_{(\Lambda)}\times\mathbf{E}\right), \tag{25}$$

$$\boldsymbol{\nabla}\cdot\mathbf{E} = -2\boldsymbol{\kappa}\cdot\mathbf{E}, \tag{26}$$

$$-\frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t} + \boldsymbol{\nabla}\times\mathbf{B} = 2\left(\frac{1}{c}\kappa_0\mathbf{E} + \boldsymbol{\kappa}\times\mathbf{B}\right). \tag{27}$$

The homogeneous currents vanish, for example, when both Λ-based wave numbers are zero. Another case where they vanish is when $\boldsymbol{\kappa}_{(\Lambda)}$ is parallel to **E**, and **B** is zero.

In the ECE2 variant of ECE theory [2], it is possible to interpret the spin connections as special potentials, called Φ_W and **W**. Because we have two types

[1] The spin connections and their differences result from the derivation of the field equations in tensor form. The details cannot be presented here, but they can be found in ref. [3], Chapter 4.

of spin connections (with and without Λ), we have two types of W potentials (scalar and vector):

$$\Phi_{(\Lambda)W} = cW^{(0)}\omega^0_{(\Lambda)}, \tag{28}$$

$$\mathbf{W}_{(\Lambda)} = W^{(0)}\boldsymbol{\omega}_{(\Lambda)}, \tag{29}$$

$$\Phi_W = cW^{(0)}\omega^0, \tag{30}$$

$$\mathbf{W} = W^{(0)}\boldsymbol{\omega}. \tag{31}$$

Then, the definitions of the κ's, Eqs. (20-23), take the following form:

$$\kappa_{(\Lambda)0} = \frac{1}{cW^{(0)}}\left(\phi - \phi_{(\Lambda)W}\right), \tag{32}$$

$$\boldsymbol{\kappa}_{(\Lambda)} = \frac{1}{W^{(0)}}\left(\mathbf{A} - \mathbf{W}_{(\Lambda)}\right), \tag{33}$$

$$\kappa_0 = \frac{1}{cW^{(0)}}\left(\phi - \phi_W\right), \tag{34}$$

$$\boldsymbol{\kappa} = \frac{1}{W^{(0)}}\left(\mathbf{A} - \mathbf{W}\right). \tag{35}$$

The Λ-based potentials play a role only for the homogeneous currents. These currents vanish if

$$\phi = \phi_{(\Lambda)W} \quad \text{and} \tag{36}$$

$$\mathbf{A} = \mathbf{W}_{(\Lambda)}. \tag{37}$$

Consequently, the Λ-based potentials do not occur in standard theory. In ECE theory, electric charges and currents are based on the potentials ϕ_W and $\mathbf{W}$, in addition to the "standard" potentials ϕ and $\mathbf{A}$. In free space, these potentials are identical to their W counterparts:

$$\phi = \phi_W \quad \text{and} \tag{38}$$

$$\mathbf{A} = \mathbf{W}. \tag{39}$$

In this case, we have an electromagnetic field without charges and currents, and ECE theory coincides with standard theory:

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0, \tag{40}$$

$$\frac{\partial \mathbf{B}}{\partial t} + \boldsymbol{\nabla} \times \mathbf{E} = \mathbf{0}, \tag{41}$$

$$\boldsymbol{\nabla} \cdot \mathbf{E} = 0, \tag{42}$$

$$-\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \boldsymbol{\nabla} \times \mathbf{B} = \mathbf{0}. \tag{43}$$

2.2 Interpretation of the current terms

2.2.1 Interpretation of standard current and charge density

Now we will try to find interpretations for the current terms in Eqs. (24-27). The most evident example is the term containing **E** on the right side of (27). In electrical engineering, it is often assumed that the electrical current is proportional to the electric field vector. The constant connecting both is the conductivity σ:

$$\mathbf{J}_E = \sigma \mathbf{E}. \tag{44}$$

This constant is an empirical extension of Maxwell's equations. Comparing Eq. (44)with Eq. (27), we see that this constant can be written in the form:

$$\sigma = \frac{2}{\mu_0 c} \kappa_0, \tag{45}$$

which is the most general corresponding conductivity term in ECE theory.

It is known from experiment that the conductivity is frequency dependent. This is in accordance with the result of ECE theory, because κ_0 is a spatial frequency that is connected to a time frequency ω_t via

$$\omega_t = c\kappa_0. \tag{46}$$

The term $\boldsymbol{\kappa} \times \mathbf{B}$ in (27) is a magnetic contribution to the current that looks like the Lorentz force. To investigate this, we first write the empirical conductivity expression (44) for a case where a magnetic field is present [4]. Then, this field produces a Lorentz force term that contributes to the electric current density in the form:

$$\mathbf{J}_E = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}). \tag{47}$$

The Lorentz force on a charge q moving with velocity $\mathbf{v}$ (including the electrical part) is

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \tag{48}$$

wich can be written for an infinitesimally small charge in the form:

$$d\mathbf{F} = dq(\mathbf{E} + \mathbf{v} \times \mathbf{B}). \tag{49}$$

Dividing this by the corresponding infinitesimal volume element, we obtain the Lorentz force density:

$$\mathbf{F}_0 = \rho(\mathbf{E} + \mathbf{v} \times \mathbf{B}). \tag{50}$$

The current density can be written as the motion of a charge continuum ρ with velocity $\mathbf{v}$:

$$\mathbf{J}_E = \rho \mathbf{v}. \tag{51}$$

Therefore, the total Lorentz force density is

$$\mathbf{F}_0 = \rho \mathbf{E} + \mathbf{J}_E \times \mathbf{B}, \tag{52}$$

and the Lorentz force for the continuum charge is

$$\mathbf{F} = \int \mathbf{F}_0 dV. \tag{53}$$

Comparing Eqs. (47) and (50), it follows that

$$\mathbf{J}_E = \frac{\sigma}{\rho}\mathbf{F}_0. \tag{54}$$

Now, we can find an expression for $\boldsymbol{\kappa}$ in Eq. (27), which will show that the term $\boldsymbol{\kappa} \times \mathbf{B}$ actually describes a Lorentz force density. By equating

$$\mathbf{J}_E = \mathbf{J}, \tag{55}$$

and respecting the factor μ_0 in (19), we find that

$$\mu_0\, \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = 2\left(\frac{1}{c}\kappa_0\mathbf{E} + \boldsymbol{\kappa} \times \mathbf{B}\right). \tag{56}$$

Comparing the second terms on both sides, it follows that

$$\mu_0\sigma\mathbf{v} = 2\boldsymbol{\kappa}, \tag{57}$$

or, with Eq. (45),

$$\boldsymbol{\kappa} = \frac{\kappa_0}{c}\mathbf{v}. \tag{58}$$

Thus, we have shown that Eq. (27) contains the Lorentz force density in a non-relativistic approximation. While the term $\sigma\mathbf{E}$ represents a laminar flow, the Lorentz force term is rotational and leads to a turbulent flow.

As an example, we investigate the Coulomb law, Eq. (26):

$$\boldsymbol{\nabla} \cdot \mathbf{E} = -2\boldsymbol{\kappa} \cdot \mathbf{E}. \tag{59}$$

For a point charge q, the electric field depends only on the radial coordinate:

$$\mathbf{E} = E_r\hat{\mathbf{r}} = \frac{q}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}}, \tag{60}$$

where $\hat{\mathbf{r}}$ is the unit vector in the radial direction. The divergence of $\mathbf{E}$ is

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\partial}{\partial r}E_r = -2\frac{q}{4\pi\epsilon_0 r^3}. \tag{61}$$

Equating this with the right side of Eq. (59), we obtain

$$-2\kappa_r E_r = -2\kappa_r \frac{q}{4\pi\epsilon_0 r^2} = -2\frac{q}{4\pi\epsilon_0 r^3}, \tag{62}$$

from which it follows that the radial part of the wavevector $\boldsymbol{\kappa}$ is

$$\kappa_r = \frac{1}{r}. \tag{63}$$

Except for constants, κ_r is identical to the Coulomb potential.

2.2.2 Interpretation of the homogeneous current and associated charge density

We now come to the homogeneous current – the charge and current terms of the Gauss (5) and Faraday (6) laws. Similar as before, we start from Equations (24) and (25), which contain the homogeneous terms in wave vector notation.

The Gauss law (24),

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 2\boldsymbol{\kappa}_{(\Lambda)} \cdot \mathbf{B}, \tag{64}$$

now has a term on the right side analogous to the Coulomb law (26). This term corresponds to a magnetic charge density. As in standard theory, we start by assuming that

$$\boldsymbol{\kappa}_{(\Lambda)} = \mathbf{0}. \tag{65}$$

Please note that this wave number is different from that in the Coulomb law. If the scalar wave number $\kappa_{(\Lambda)0}$ also vanishes, then the entire homogeneous current in the Faraday law,

$$\frac{\partial \mathbf{B}}{\partial t} + \boldsymbol{\nabla} \times \mathbf{E} = 2\left(c\kappa_{(\Lambda)0}\mathbf{B} - \boldsymbol{\kappa}_{(\Lambda)} \times \mathbf{E}\right), \tag{66}$$

vanishes, and we return to the situation in standard theory.

The homogeneous current of the Faraday law is formally dual to the inhomogeneous (standard) current in the Ampère-Maxwell law (27). All **E** and **B** terms are interchanged on both sides of the equations[2]. Using the same argumentation as above, we find that $2c\kappa_{(\Lambda)0}\mathbf{B}$ is a magnetic conductivity term, while $-\boldsymbol{\kappa}_{(\Lambda)} \times \mathbf{E}$ is the inverse Lorentz force density [5], which in standard notation is

$$\mathbf{B} = -\frac{1}{c^2}\mathbf{v} \times \mathbf{E}. \tag{67}$$

In the following, we work this out in detail.

The term $-\boldsymbol{\kappa}_{(\Lambda)} \times \mathbf{E}$ in (66) is an electric contribution to the homogeneous current that looks like the inverse Lorentz force. We define an expression for a magnetic conductivity σ_h in ananlogy to Eq. (47). Then, we obtain a conductivity that originates in a magnetic field (instead of an electric field) and contains an additional term of the inverse Lorentz force in the form:

$$\mathbf{J}_B = \sigma_h(\mathbf{B} - \frac{1}{c^2}\mathbf{v} \times \mathbf{E}). \tag{68}$$

In full analogy to the previous derivation, we obtain an inverse Lorentz force density:

$$\mathbf{F}_{0h} = \rho_h(\mathbf{B} - \frac{1}{c^2}\mathbf{v} \times \mathbf{E}), \tag{69}$$

where ρ_h is the density of "magnetic charges".

The current density can again be written as the motion of a charge continuum ρ_h with velocity **v**:

$$\mathbf{J}_B = \rho_h\mathbf{v}. \tag{70}$$

[2]The deeper reason is that Eq. (25) is the Hodge dual of Eq. (27), which is a property of Cartan geometry.

Therefore, the density of the inverse Lorentz force is

$$\mathbf{F}_{0h} = \rho_h \mathbf{B} - \frac{1}{c^2}\mathbf{J}_B \times \mathbf{E}, \tag{71}$$

and the total inverse Lorentz force is

$$\mathbf{F}_h = \int \mathbf{F}_{0h} dV. \tag{72}$$

Comparing Eqs. (68) and (71), it follows that

$$\mathbf{J}_B = \frac{\sigma_h}{\rho_h}\mathbf{F}_{0h}. \tag{73}$$

Now, we can find an expression for $\boldsymbol{\kappa}_{(\Lambda)}$ in Eq. (25) which will show that the term $-\boldsymbol{\kappa}_{(\Lambda)} \times \mathbf{E}$ actually describes an inverse Lorentz force density. By equating

$$\mathbf{J}_B = \mathbf{j}, \tag{74}$$

and respecting the factor $c\mu_0$ in (17), we find that

$$c\mu_0\, \sigma_h(\mathbf{B} - \frac{1}{c^2}\mathbf{v} \times \mathbf{E}) = 2\left(c\kappa_0\mathbf{B} - \boldsymbol{\kappa}_{(\Lambda)} \times \mathbf{E}\right). \tag{75}$$

In analogy to Eq. (45), we obtain

$$\sigma_h = \frac{2\kappa_0}{\mu_0}. \tag{76}$$

Comparing the second terms on both sides, it follows that

$$\frac{\mu_0\sigma_h}{c}\mathbf{v} = 2\boldsymbol{\kappa}_{(\Lambda)}. \tag{77}$$

or

$$\boldsymbol{\kappa}_{(\Lambda)} = \frac{\kappa_{(\Lambda)0}}{c}\mathbf{v}. \tag{78}$$

Thus, we have shown that Eq. (25) contains the inverse Lorentz force density in a non-relativistic approximation. While the term $\sigma_h\mathbf{B}$ represents a laminar flow, the Lorentz force term is rotational and leads to a turbulent flow.

Quantity	Unit
μ_0	$\frac{Vs}{Am}$
σ	$\frac{A}{Vm}$
σ_h	$\frac{A}{Vs}$
$\mathbf{J}$	$\frac{A}{m^2}$
$\mathbf{j}$	$\frac{A}{ms}$
$\mu_0\mathbf{J}$	$\frac{Vs}{m^3} = \frac{T}{m}$
$c\mu_0\mathbf{j}$	$\frac{V}{m^2}$

Table 1: Physical units of conductivity and current quantities.

In Table 1, the physical units of the conductivity and current terms of both the inhomogeneous and homogeneous current are listed for comparison. The homogeneous current will further be discussed in the subsequent paper.

2.2.3 Wave equation with the homogeneous current

In standard electrodynamics, we have wave equations for the electric and magnetic fields, which indicates that wave propagation of fields is possible. We extend this to include the homogeneous current. Applying the time derivative to Eq. (8) gives

$$-\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} + \boldsymbol{\nabla} \times \frac{\partial \mathbf{B}}{\partial t} = \mu_0 \frac{\partial \mathbf{J}}{\partial t}, \tag{79}$$

and taking the curl of Eq. (6) leads to

$$\boldsymbol{\nabla} \times \frac{\partial \mathbf{B}}{\partial t} + \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{E} = c\,\mu_0\, \boldsymbol{\nabla} \times \mathbf{j}. \tag{80}$$

After replacing the double-curl according to the rule

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{E} = -\nabla^2 \mathbf{E} + \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{E}), \tag{81}$$

and assuming no charge density ($\boldsymbol{\nabla} \cdot \mathbf{E} = 0$), we can insert (80) into (79) to obtain

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = -\mu_0 \frac{\partial \mathbf{J}}{\partial t} + \mu_0 \boldsymbol{\nabla} \times \mathbf{j}. \tag{82}$$

This is the wave equation for $\mathbf{E}$ with two inhomogeneous terms, consisting of the inhomogeneous and homogeneous currents. In normal free space conditions, both are zero. Furthermore, both vanish when $\mathbf{J}$ is time-independent and $\mathbf{j}$ is curl-free.

In magnetostatics we have

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0\, \mathbf{J}. \tag{83}$$

If a homogeneous current is present, Eq. (6) gives the following result for electrostatics:

$$\boldsymbol{\nabla} \times \mathbf{E} = c\mu_0\, \mathbf{j}. \tag{84}$$

We again see the complete duality between electric and magnetic fields, including both types of currents. In the second paper of this series, we will further investigate the properties and effects of the homogeneous current, including polarization of the vacuum.

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References

[1] M. W. Evans, “Generally Covariant Unified Field Theory: The Geometrization of Physics”, Vols. 1 to 7, Abramis Academic, Bury St Edmunds, 2005 ff.

[2] H. Eckardt, “ECE UFT - The Geometrical Basis of Physics, Vol. I - Classical Physics”, textbook, epubli, Berlin, 2022, freely available as UFT Paper 438, Unified Field Theory (UFT) section of www.aias.us.

[3] H. Eckardt, “ECE UFT - The Geometrical Basis of Physics, Vol. II - Quantum Physics”, textbook, in preparation, UFT Paper 448, Unified Field Theory (UFT) section of www.aias.us.

[4] M. W. Evans, H. Eckardt, Paper 349 and related notes, Unified Field Theory (UFT) Section of www.aias.us.

[5] J. D. Jackson, “Classical Electrodynamics”, Section 11.10, Third Edition, John Wiley and Sons, 1998.