PRINCIPLES OF ECE THEORY

A NEW PARADIGM OF PHYSICS

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This book is dedicated to
all wholehearted scholars of natural philosophy

Contents

Chapter 1

ECE Theory and Beltrami Fields

1.1 Introduction

Towards the end of the nineteenth century the Italian mathematician Eugenio Beltrami developed a system of equations for the description of hydrodynamic flow in which the curl of a vector is proportional to the vector itself. An example is the use of the velocity vector. For a long time this solution was not used outside the field of hydrodynamics, but in the fifties it started to be used by workers such as Alfven and Chandrasekhar in the area of cosmology, notably whirlpool galaxies. The Beltrami field as it came to be known has been observed in plasma vortices and as argued by Reed [1] is indicative of a type of electrodynamics such as ECE. Therefore this chapter is concerned with the ways in which ECE electrodynamics reduce to Beltrami electrodynamics, and with other applications of the Beltrami electrodynamics such as a new theory of the parton structure of elementary particles. The ECE theory is based on geometry and is ubiquitous throughout nature on all scales, and so is the Beltrami theory, which can be looked upon as a sub theory of ECE theory.

1.2 Derivation of the Beltrami Equation

Consider the Cartan identity in vector notation, derived in Chapter 2:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\omega}^a{}_b \times \mathbf{q}^b = \mathbf{q}^b \cdot \boldsymbol{\nabla} \times \boldsymbol{\omega}^a{}_c - \boldsymbol{\omega}^a{}_b \cdot \boldsymbol{\nabla} \times \mathbf{q}^b. \tag{1.1}$$

In the absence of a magnetic monopole:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\omega}^a{}_b \times \mathbf{q}^b = 0 \tag{1.2}$$

so:

$$\mathbf{q}^b \cdot \boldsymbol{\nabla} \times \boldsymbol{\omega}^a{}_b = \boldsymbol{\omega}^a{}_b \cdot \boldsymbol{\nabla} \times \mathbf{q}^b. \tag{1.3}$$

Assume that the spin connection is an axial vector dual in its index space to an antisymmetric tensor:

$$\boldsymbol{\omega}^a{}_b = \epsilon^a{}_{bc} \boldsymbol{\omega}^c \tag{1.4}$$

where $\epsilon^a{}_{bc}$ is the totally antisymmetric unit tensor in three dimensions. Then Eq. (1.3) reduces to:

$$\mathbf{q}^b \cdot \boldsymbol{\nabla} \times \boldsymbol{\omega}^c = \boldsymbol{\omega}^c \cdot \boldsymbol{\nabla} \times \mathbf{q}^b. \tag{1.5}$$

An example of this in electromagnetism is:

$$\mathbf{A}^{(2)} \cdot \boldsymbol{\nabla} \times \boldsymbol{\omega}^{(1)} = \boldsymbol{\omega}^{(1)} \cdot \boldsymbol{\nabla} \times \mathbf{A}^{(2)} \tag{1.6}$$

in the complex circular basis ((1), (2), (3)). The vector potential is defined by the ECE hypothesis:

$$\mathbf{A}^a = A^{(0)} \mathbf{q}^a. \tag{1.7}$$

From Chap. 2, Eq, (2.76) the geometrical condition for the absence of a magnetic monopole is:

$$\boldsymbol{\omega}^a{}_b \cdot \mathbf{B}^b = \mathbf{A}^b \cdot \mathbf{R}^a{}_b(\text{spin}) \tag{1.8}$$

where the spin curvature Eq. (2.63) is defined by:

$$\mathbf{R}^a{}_b(\text{spin}) = \boldsymbol{\nabla} \times \boldsymbol{\omega}^a{}_b - \boldsymbol{\omega}^a{}_c \times \boldsymbol{\omega}^c{}_b \tag{1.9}$$

and where $\mathbf{B}^a$ is the magnetic flux density vector. Using Eq. (1.4):

$$\mathbf{R}^c(\text{spin}) = \boldsymbol{\nabla} \times \boldsymbol{\omega}^c - \boldsymbol{\omega}^b \times \boldsymbol{\omega}^a. \tag{1.10}$$

In the complex circular basis defined by Eq. (1.6) the spin curvatures are:

$$\begin{aligned}
\mathbf{R}^{(1)}(\text{spin}) &= \boldsymbol{\nabla} \times \boldsymbol{\omega}^{(1)} + i\, \boldsymbol{\omega}^{(3)} \times \boldsymbol{\omega}^{(1)} \\
\mathbf{R}^{(2)}(\text{spin}) &= \boldsymbol{\nabla} \times \boldsymbol{\omega}^{(2)} + i\, \boldsymbol{\omega}^{(2)} \times \boldsymbol{\omega}^{(3)} \\
\mathbf{R}^{(3)}(\text{spin}) &= \boldsymbol{\nabla} \times \boldsymbol{\omega}^{(3)} + i\, \boldsymbol{\omega}^{(1)} \times \boldsymbol{\omega}^{(2)}
\end{aligned} \tag{1.11}$$

and the magnetic flux density vectors are:

$$\begin{aligned}\mathbf{B}^{(1)} &= \boldsymbol{\nabla} \times \mathbf{A}^{(1)} + i\,\boldsymbol{\omega}^{(3)} \times \mathbf{A}^{(1)} \\ \mathbf{B}^{(2)} &= \boldsymbol{\nabla} \times \mathbf{A}^{(2)} + i\,\boldsymbol{\omega}^{(2)} \times \mathbf{A}^{(2)} \\ \mathbf{B}^{(3)} &= \boldsymbol{\nabla} \times \mathbf{A}^{(3)} + i\,\boldsymbol{\omega}^{(1)} \times \mathbf{A}^{(3)}.\end{aligned} \tag{1.12}$$

Eq. (8) may be exemplified by:

$$\boldsymbol{\omega}^{(1)} \cdot \mathbf{B}^{(2)} = \mathbf{A}^{(1)} \cdot \mathbf{R}^{(2)}(\text{spin}) \tag{1.13}$$

which may be developed as:

$$\begin{aligned}&\boldsymbol{\omega}^{(1)} \cdot \left(\boldsymbol{\nabla} \times \mathbf{A}^{(2)} + i\,\boldsymbol{\omega}^{(2)} \times \mathbf{A}^{(3)}\right) \\ &= \mathbf{A}^{(1)} \cdot \left(\boldsymbol{\nabla} \times \boldsymbol{\omega}^{(2)} + i\,\boldsymbol{\omega}^{(2)} \times \boldsymbol{\omega}^{(3)}\right).\end{aligned} \tag{1.14}$$

Possible solutions are

$$\boldsymbol{\omega}^{(i)} = \pm\frac{\kappa}{A^{(0)}}\mathbf{A}^{(i)},\ \mathrm{i} = 1, 2, 3 \tag{1.15}$$

and in order to be consistent with the original [1-10] solution of B(3) the negative sign is developed:

$$\mathbf{B}^{(3)} = \boldsymbol{\nabla} \times \mathbf{A}^{(3)} - i\frac{\kappa}{A^{(0)}}\mathbf{A}^{(1)} \times \mathbf{A}^{(2)} \text{ et cyclicum.} \tag{1.16}$$

From Eq. (3.2):

$$\boldsymbol{\nabla} \cdot \boldsymbol{\omega}^{(3)} \times \mathbf{A}^{(1)} = 0 \tag{1.17}$$

and the following is an identity of vector analysis:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} \times \mathbf{A}^{(1)} = 0. \tag{1.18}$$

A possible solution of Eq. (3.17) is:

$$\boldsymbol{\nabla} \times \mathbf{A}^{(1)} = i\,\boldsymbol{\omega}^{(3)} \times \mathbf{A}^{(1)} = -i\,\frac{\kappa}{A^{(0)}}\mathbf{A}^{(3)} \times \mathbf{A}^{(1)}. \tag{1.19}$$

Similarly:

$$\boldsymbol{\nabla} \times \mathbf{A}^{(2)} = i\,\boldsymbol{\omega}^{(2)} \times \mathbf{A}^{(3)} = -i\,\frac{\kappa}{A^{(0)}}\mathbf{A}^{(2)} \times \mathbf{A}^{(3)}. \tag{1.20}$$

Now multiply both sides of the basis equations (3.6) to (3.8) of Chap. 2 by

$$A^{(0)2}\ \mathrm{e}^{i\phi}\ \mathrm{e}^{-i\phi} \tag{1.21}$$

where the electromagnetic phase is:

$$\phi = \omega t - \kappa Z \tag{1.22}$$

to find the cyclic equation:

$$\mathbf{A}^{(1)} \times \mathbf{A}^{(2)} = i\ A^{(0)}\mathbf{A}^{(3)*} \text{ et cyclicum} \tag{1.23}$$

where:

$$\mathbf{A}^{(1)} = \mathbf{A}^{(2)*} = A^{(0)}\mathbf{e}^{(1)}\mathrm{e}^{i\phi} = \frac{A^{(0)}}{\sqrt{2}}\left(\mathbf{i} - i\mathbf{j}\right)\mathrm{e}^{i\phi}, \tag{1.24}$$

$$\mathbf{A}^{(3)} = A^{(0)}\mathbf{e}^{(3)} = A^{(0)}\mathbf{k}. \tag{1.25}$$

From Eqs. (1.23-1.25):

$$\boldsymbol{\nabla} \times \mathbf{A}^{(1)} = \kappa \mathbf{A}^{(1)} \tag{1.26}$$

$$\boldsymbol{\nabla} \times \mathbf{A}^{(2)} = \kappa \mathbf{A}^{(2)} \tag{1.27}$$

$$\boldsymbol{\nabla} \times \mathbf{A}^{(3)} = 0\mathbf{A}^{(3)} \tag{1.28}$$

which are Beltrami equations [1]. The foregoing analysis may be simplified by considering only one component out of the two conjugate components labelled (1) and (2). This procedure, however, loses information in general. By considering one component, Eq. (1.1) is simplified to:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\omega} \times \mathbf{q} = \mathbf{q} \cdot \boldsymbol{\nabla} \times \boldsymbol{\omega} - \boldsymbol{\omega} \cdot \boldsymbol{\nabla} \times \mathbf{q} \tag{1.29}$$

and the assumption of zero magnetic monopole leads to:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\omega} \times \mathbf{q} = 0 \tag{1.30}$$

which implies

$$\boldsymbol{\omega} \cdot \boldsymbol{\nabla} \times \mathbf{q} = \mathbf{q} \cdot \boldsymbol{\nabla} \times \boldsymbol{\omega}. \tag{1.31}$$

Proceeding as in note 257(7) in the UFT section of www.aias.us leads to:

$$\boldsymbol{\omega} \cdot \mathbf{B} = \mathbf{A} \cdot \boldsymbol{\nabla} \times \boldsymbol{\omega} \tag{1.32}$$

where:

$$\mathbf{R}(\text{spin}) = \boldsymbol{\nabla} \times \boldsymbol{\omega} \tag{1.33}$$

is the simplified format of the spin curvature. From Eqs. (1.31) and (1.32):

$$\boldsymbol{\omega} \cdot \mathbf{B} = \mathbf{A} \cdot \boldsymbol{\nabla} \times \boldsymbol{\omega} = \boldsymbol{\omega} \cdot \boldsymbol{\nabla} \times \mathbf{A} \tag{1.34}$$

so:

$$\mathbf{B} = \mathbf{A} \cdot \boldsymbol{\nabla} \times \mathbf{A}. \tag{1.35}$$

However, in ECE theory:

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A} - \boldsymbol{\omega} \times \mathbf{A} \tag{1.36}$$

so Eqs. (1.35) and (1.36) imply:

$$\boldsymbol{\omega} \times \mathbf{A} = \mathbf{0}. \tag{1.37}$$

Therefore in this simplified model the spin connection vector is parallel to the vector potential. These results are consistent with [1-10]:

$$p^\mu = eA^\mu = \hbar\kappa^\mu = \hbar\omega^\mu \tag{1.38}$$

from the minimal prescription. So in this simplified model:

$$\omega^\mu = (\omega_0, \boldsymbol{\omega}) = \frac{e}{\hbar}A^\mu = \frac{e}{\hbar}(A_0, \mathbf{A}). \tag{1.39}$$

The electric field strength is defined in the simplified model by:

$$\mathbf{E} = -\boldsymbol{\nabla}\phi - \frac{\partial \mathbf{A}}{\partial t} - c\omega_0\mathbf{A} + \phi\boldsymbol{\omega} \tag{1.40}$$

where the scalar potential is

$$\phi = cA_0. \tag{1.41}$$

From Eqs. (3.39) and (3.40):

$$\mathbf{E} = -\boldsymbol{\nabla}\phi - \frac{\partial \mathbf{A}}{\partial t}, \tag{1.42}$$

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}, \tag{1.43}$$

which is the same as the structure given by Heaviside, but these equations have been derived from general relativity and Cartan geometry, whereas the Heaviside structure is empirical. The equations (1.29) to (1.43) are oversimplified however because they are derived by consideration of only one out of two conjugate conjugates (1) and (2). Therefore they are derived using real algebra instead of complex algebra. They lose the B(3) field and also spin connection resonance, developed later in this book.

In the case of field matter interaction the electric field strength, $\mathbf{E}$ is replaced by the electric displacement, $\mathbf{D}$, and the magnetic flux density, $\mathbf{B}$ by the magnetic field strength, $\mathbf{H}$:

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}, \tag{1.44}$$

$$\mathbf{H} = \frac{1}{\mu_0}(\mathbf{B} - \mathbf{M}), \tag{1.45}$$

where $\mathbf{P}$ is the polarization, $\mathbf{M}$ is the magnetization, ϵ_0 is the vacuum permittivity and μ_0 is the vacuum permeability. The four equations of electrodynamics for each index (1) or (2) are:

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \tag{1.46}$$

$$\boldsymbol{\nabla} \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0} \tag{1.47}$$

$$\boldsymbol{\nabla} \cdot \mathbf{D} = \rho \tag{1.48}$$

$$\boldsymbol{\nabla} \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \tag{1.49}$$

where ρ is the charge density and $\mathbf{J}$ is the current density.

The Gauss law of magnetism:

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \tag{1.50}$$

implies the magnetic Beltrami equation [1]:

$$\boldsymbol{\nabla} \times \mathbf{B} = \kappa \mathbf{B} \tag{1.51}$$

because:

$$\frac{1}{\kappa} \boldsymbol{\nabla} \cdot \boldsymbol{\nabla} \times \mathbf{B} = 0. \tag{1.52}$$

So the magnetic Beltrami equation is a consequence of the absence of a magnetic monopole and the Beltrami solution is always a valid solution. From Eqs. (1.49) and (1.2)

$$\boldsymbol{\nabla} \times \mathbf{B} = \kappa \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \tag{1.53}$$

and for magnetostatics or if the Maxwell displacement current is small:

$$\mathbf{B} = \frac{\mu_0}{\kappa} \mathbf{J}. \tag{1.54}$$

In this case the magnetic flux density is proportional to the current density. From Eq. :

$$\boldsymbol{\nabla} \times \mathbf{B} = \frac{\mu_0}{\kappa} \boldsymbol{\nabla} \times \mathbf{J} = \kappa \mathbf{B} \tag{1.55}$$

so

$$\mathbf{B} = \frac{\mu_0}{\kappa^2} \boldsymbol{\nabla} \times \mathbf{J}. \tag{1.56}$$

Eqs. and imply that the current density must have the structure:

$$\boldsymbol{\nabla} \times \mathbf{J} = \kappa \mathbf{J} \tag{1.57}$$

in order to produce a Beltrami equation in magnetostatics. Eq. suggests that the jet observed from the plane of a whirlpool galaxy is a longitudinal solution of the Beltrami equation, a J(3) current associated with a B(3) field.

In field matter interaction the electric Beltrami equation:

$$\boldsymbol{\nabla} \times \mathbf{E} = \kappa \mathbf{E} \tag{1.58}$$

is not valid because it is not consistent with the Coulomb law:

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}. \tag{1.59}$$

From Eqs. (1.58) and (1.59):

$$\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} \times \mathbf{E} = \frac{\rho}{\epsilon_0} \kappa \tag{1.60}$$

which violates the vector identity:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} \times \mathbf{E} = 0. \tag{1.61}$$

The electric Beltrami equation:

$$\boldsymbol{\nabla} \times \mathbf{E} = \kappa \mathbf{E} \tag{1.62}$$

is valid for the free electromagnetic field.

Consider the four equations of the free electromagnetic field:

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \tag{1.63}$$

$$\boldsymbol{\nabla} \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0} \tag{1.64}$$

$$\boldsymbol{\nabla} \cdot \mathbf{E} = 0 \tag{1.65}$$

$$\boldsymbol{\nabla} \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mathbf{0} \tag{1.66}$$

for each index of the complex circular basis. It follows from Eqs. (1.64) and (1.66) that:

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{B}) = \frac{1}{c^2}\frac{\partial}{\partial t}\boldsymbol{\nabla} \times \mathbf{E} \tag{1.67}$$

and:

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{E}) = -\frac{\partial}{\partial t}\boldsymbol{\nabla} \times \mathbf{B}. \tag{1.68}$$

The transverse plane wave solutions are:

$$\mathbf{E} = \frac{E^{(0)}}{\sqrt{2}}\left(\mathbf{i} - i\mathbf{j}\right)e^{i\phi} \tag{1.69}$$

and

$$\mathbf{B} = \frac{B^{(0)}}{\sqrt{2}}\left(i\mathbf{i} + \mathbf{j}\right)e^{i\phi} \tag{1.70}$$

where:

$$\phi = \omega t - \kappa Z \tag{1.71}$$

and where ω is the angular velocity at instant t and κ is the magnitude of the wave vector at Z.

From vector analysis:

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{B}) = \boldsymbol{\nabla}\left(\boldsymbol{\nabla} \cdot \mathbf{B}\right) - \boldsymbol{\nabla}^2\mathbf{B} \tag{1.72}$$

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{E}) = \boldsymbol{\nabla}\left(\boldsymbol{\nabla} \cdot \mathbf{E}\right) - \boldsymbol{\nabla}^2\mathbf{E} \tag{1.73}$$

and for the free field the divergences vanish, so we obtain the Helmholtz wave equations:

$$(\boldsymbol{\nabla}^2 + \kappa^2)\mathbf{B} = 0 \tag{1.74}$$

and

$$(\boldsymbol{\nabla}^2 + \kappa^2)\mathbf{E} = 0. \tag{1.75}$$

These are the Trkalian equations:

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{B}) = \kappa\boldsymbol{\nabla} \times \mathbf{B} = \kappa^2\mathbf{B} \tag{1.76}$$

and

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{E}) = \kappa \boldsymbol{\nabla} \times \mathbf{E} = \kappa^2 \mathbf{E}. \tag{1.77}$$

So solutions of the Beltrami equations are also solutions of the Helmholtz wave equations. From Eqs. (1.64), (1.67) and (1.76):

$$-\boldsymbol{\nabla}^2 \mathbf{B} - \frac{\kappa}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \left(-\boldsymbol{\nabla}^2 + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \mathbf{B} = 0 \tag{1.78}$$

which is the d'Alembert equation:

$$\Box \, \mathbf{B} = \mathbf{0}. \tag{1.79}$$

For finite photon mass, implied by the longitudinal solutions of the free electromagnetic field:

$$\hbar^2 \omega^2 = c^2 \hbar^2 \kappa^2 + m_0^2 c^4 \tag{1.80}$$

in which case:

$$\left(\Box + \left(\frac{m_0 c}{\hbar} \right)^2 \right) \mathbf{B} = \mathbf{0} \tag{1.81}$$

which is the Proca equation. This was first derived in ECE theory from the tetrad postulate of Cartan geometry and is discussed later in this book. From Eqs. (1.67) and (1.68):

$$\frac{\partial^2}{\partial t^2} \boldsymbol{\nabla} \times \mathbf{B} = -\omega^2 \boldsymbol{\nabla} \times \mathbf{B} \tag{1.82}$$

and:

$$\frac{\partial^2}{\partial t^2} \boldsymbol{\nabla} \times \mathbf{E} = -\omega^2 \boldsymbol{\nabla} \times \mathbf{E}. \tag{1.83}$$

In general:

$$\frac{\partial^2}{\partial t^2} e^{i\phi} = -\omega^2 e^{i\phi} \tag{1.84}$$

and

$$e^{i\phi} = e^{i\omega t} e^{-i\kappa Z} \tag{1.85}$$

so the general solution of the Beltrami equation

$$\boldsymbol{\nabla} \times \mathbf{B} = \kappa \mathbf{B} \tag{1.86}$$

will also be a general solution of the equations (1.63) to (1.66) multiplied by the phase factor $\exp(i\omega t)$.

ECE theory can be used to show that the magnetic flux density, vector potential and spin connection vector are always Beltrami vectors with intricate structures in general, solutions of the Beltrami equation. The Beltrami structure of the vector potential is proven in ECE physics from the Beltrami structure of the magnetic flux density **B**. The space part of the Cartan identity also has a Beltrami structure. If real algebra is used, the Beltrami structure of **B** immediately refutes U(1) gauge invariance because **B** becomes directly proportional to **A**. It follows that the photon mass is identically non-zero, however tiny in magnitude. Therefore there is no Higgs boson in nature because the latter is the result of U(1) gauge invariance. The Beltrami structure of **B** is the direct result of the Gauss law of magnetism and the absence of a magnetic monopole. It is difficult to conceive why U(1) gauge invariance should ever have been adopted as a theory, because its refutation is trivial. Once U(1) gauge invariance is discarded a rich panoply of new ideas and results emerge.

The Beltrami equation for magnetic flux density in ECE physics is:

$$\boldsymbol{\nabla} \times \mathbf{B}^a = \kappa \mathbf{B}^a. \tag{1.87}$$

In the simplest case κ is a wave-vector but it can become very intricate. Combining Eq. (1.87) with the Ampere Maxwell law of ECE physics:

$$\boldsymbol{\nabla} \times \mathbf{B}^a = \mu_0 \mathbf{J}^a + \frac{1}{c^2}\frac{\partial \mathbf{E}^a}{\partial t} \tag{1.88}$$

the magnetic flux density is given directly by:

$$\mathbf{B}^a = \frac{1}{\kappa}\left(\frac{1}{c^2}\frac{\partial \mathbf{E}^a}{\partial t} + \mu_0 \mathbf{J}^a\right). \tag{1.89}$$

Using the Coulomb law of ECE physics:

$$\boldsymbol{\nabla} \cdot \mathbf{E}^a = \frac{\rho^a}{\epsilon_0} \tag{1.90}$$

it is found that:

$$\boldsymbol{\nabla} \cdot \mathbf{B}^a = \frac{\mu_0}{\kappa}\left(\frac{\partial \rho^a}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{J}^a\right) = 0, \tag{1.91}$$

a result which follows from:

$$\epsilon_0 \, \mu_0 = \frac{1}{c^2} \tag{1.92}$$

where c is the universal constant known as the vacuum speed of light. The conservation of charge current density in ECE physics is:

$$\frac{\partial \rho^a}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{J}^a = 0 \tag{1.93}$$

so $\mathbf{B}^a$ is always a Beltrami vector.

In the simplified physics with real algebra:

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}, \tag{1.94}$$

$$\boldsymbol{\nabla} \times \mathbf{B} = \kappa \boldsymbol{\nabla} \times \mathbf{A}, \tag{1.95}$$

where $\mathbf{A}$ is the vector potential. Eqs. (1.94) and (1.95) show immediately that in U(1) physics the vector potential also obeys a Beltrami equation:

$$\boldsymbol{\nabla} \times \mathbf{A} = \kappa \mathbf{A}, \tag{1.96}$$

$$\mathbf{B} = \kappa \mathbf{A} \tag{1.97}$$

so in this simplified theory the magnetic flux density is directly proportional to the vector potential $\mathbf{A}$. It follows immediately that $\mathbf{A}$ cannot be U(1) gauge invariant because U(1) gauge invariance means:

$$\mathbf{A} \rightarrow \mathbf{A} + \boldsymbol{\nabla} \psi \tag{1.98}$$

and if $\mathbf{A}$ is changed, $\mathbf{B}$ is changed. The obsolete dogma of U(1) physics asserted that Eq. (1.98) does not change any physical quantity. This dogma is obviously incorrect because $\mathbf{B}$ is a physical quantity and Eq. (1.97) changes it. Therefore there is finite photon mass and no Higgs boson. Finite photon mass and the Proca equation are developed later in this book, and the theory is summarized here for ease of reference. The Proca equation [1-10] can be developed as:

$$\boldsymbol{\nabla} \cdot \mathbf{B}^a = 0 \tag{1.99}$$

$$\boldsymbol{\nabla} \times \mathbf{E}^a + \frac{\partial \mathbf{B}^a}{\partial t} = \mathbf{0} \tag{1.100}$$

$$\boldsymbol{\nabla} \cdot \mathbf{E}^a = \frac{\rho^a}{\epsilon_0} \tag{1.101}$$

$$\boldsymbol{\nabla} \times \mathbf{B}^a - \frac{1}{c^2} \frac{\partial \mathbf{E}^a}{\partial t} = \mu_0 \mathbf{J}^a \tag{1.102}$$

where the 4-current density is:

$$J^{a\mu} = (c\rho^a, \mathbf{J}^a) \tag{1.103}$$

and where the 4-potential is:

$$A^{a\mu} = \left(\frac{\phi^a}{c}, \mathbf{A}^a\right). \tag{1.104}$$

Proca theory asserts that:

$$J^{a\mu} = -\epsilon_0 \left(\frac{m\ c}{\hbar}\right)^2 A^{a\mu} \tag{1.105}$$

where m is the finite photon mass and $\hbar$ is the reduced Planck constant. Therefore:

$$\rho^a = -\epsilon_0 c^2 \left(\frac{m\ c}{\hbar}\right)^2 \phi^a, \tag{1.106}$$

$$\mathbf{J}^a = -\epsilon_0 c^2 \left(\frac{m\ c}{\hbar}\right)^2 \mathbf{A}^a. \tag{1.107}$$

The Proca equation was inferred in the mid-thirties but is almost entirely absent from the textbooks. This is an unfortunate result of incorrect dogma, that the photon mass, is zero despite being postulated by Einstein in about 1905 to be a particle or corpuscle, as did Newton before him. The U(1) Proca theory in S. I. Units is:

$$\partial_\mu F^{\mu\nu} = \frac{J^\nu}{\epsilon_0} = -\left(\frac{m\ c}{\hbar}\right)^2 A^\nu. \tag{1.108}$$

It follows immediately that:

$$\partial_\nu \partial_\mu F^{\mu\nu} = \frac{1}{\epsilon_0} \partial_\nu J^\nu = -\left(\frac{m\ c}{\hbar}\right)^2 \partial_\nu A^\nu = 0. \tag{1.109}$$

and that:

$$\partial_\mu J^\mu = \partial_\mu A^\mu = 0. \tag{1.110}$$

Eq. (1.109) is conservation of charge current density and Eq. (1.110) is the Lorenz condition. In the Proca equation the Lorenz condition has nothing to do with gauge invariance. The U(1) gauge invariance means that:

$$A^\mu \rightarrow A^\mu + \partial^\mu \chi \tag{1.111}$$

and from Eq. (1.108) it is trivially apparent that the Proca field and charge current density change under transformation (1.111), so are not gauge invariant, QED. The entire edifice of U(1) electrodynamics collapses as soon as photon mass is considered.

In vector notation Eq. (1.109) is:

$$\frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{\nabla}\cdot\mathbf{E} = \frac{1}{c\epsilon_0}\frac{\partial \rho}{\partial t} = 0 \tag{1.112}$$

and

$$\boldsymbol{\nabla}\cdot\boldsymbol{\nabla}\times\mathbf{B} - \frac{1}{c^2}\frac{\partial}{\partial t}\boldsymbol{\nabla}\cdot\mathbf{E} = \mu_0\boldsymbol{\nabla}\cdot\mathbf{J} = 0. \tag{1.113}$$

Now use:

$$\boldsymbol{\nabla}\cdot\boldsymbol{\nabla}\times\mathbf{B} = 0 \tag{1.114}$$

and the Coulomb law of this simplified theory (without index a):

$$\boldsymbol{\nabla}\cdot\mathbf{E} = \frac{\rho}{\epsilon_0} \tag{1.115}$$

to find that:

$$-\frac{1}{c^2\epsilon_0}\frac{\partial \rho}{\partial t} = \mu_0\boldsymbol{\nabla}\cdot\mathbf{J} \tag{1.116}$$

which is the equation of charge current conservation:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}\cdot\mathbf{J} = 0. \tag{1.117}$$

In the Proca theory, Eq. (1.110) implies the Lorenz gauge as it is known in standard physics:

$$\partial_\mu A^\mu = \frac{1}{c^2}\frac{\partial \phi}{\partial t} + \boldsymbol{\nabla}\cdot\mathbf{A} = 0. \tag{1.118}$$

The Proca wave equation in the usual development [4] is obtained from the U(1) definition of the field tensor:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \tag{1.119}$$

so

$$\partial_\mu\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) = \Box A^\nu - \partial^\nu\partial_\mu A^\mu = -\left(\frac{m\,c}{\hbar}\right)^2 A^\nu \tag{1.120}$$

in which

$$\partial_\mu A^\mu = 0. \tag{1.121}$$

Eq. (1.121) follows from Eq. (1.108) in Proca physics, but in standard U(1) physics with identically zero photon mass the Lorenz gauge has to be assumed, and is arbitrary. So the Proca wave equation in the usual development [**?**] is:

$$\left(\Box + \left(\frac{m\ c}{\hbar}\right)^2\right) A^\nu = 0. \tag{1.122}$$

In ECE physics [1-10] Eq. (1.122) is derived from the tetrad postulate of Cartan geometry and becomes:

$$\left(\Box + \left(\frac{m\ c}{\hbar}\right)^2\right) A^a_\mu = 0. \tag{1.123}$$

In ECE physics the conservation of charge current density is:

$$\partial_\mu J^{a\mu} = 0 \tag{1.124}$$

and is consistent with Eqs. (1.48) and (1.49).

In ECE physics the electric charge density is geometrical in origin and is:

$$\rho^a = \epsilon_0 \left(\boldsymbol{\omega}^a{}_b \cdot \mathbf{E}^b - c\mathbf{A}^b \cdot \mathbf{R}^a{}_b(\text{orb})\right) \tag{1.125}$$

and the electric current density is:

$$\mathbf{J}^a = \frac{1}{\mu_0}\left(\boldsymbol{\omega}^a{}_b \times \mathbf{B}^b + \frac{\omega_0}{c}\mathbf{E}^b - \mathbf{A}^b \times \mathbf{R}^a{}_b(\text{spin}) - A^b{}_0\mathbf{R}^a{}_b(\text{orb})\right). \tag{1.126}$$

Here $\mathbf{R}^a{}_b(\text{spin})$ and $\mathbf{R}^a{}_b(\text{orb})$ are the spin and orbital components of the curvature tensor [1-10]. So Eqs. (1.93), (1.125) and (1.126) give many new equations of physics which can be developed systematically in future work. In magnetostatics for example the relevant equations are:

$$\boldsymbol{\nabla} \cdot \mathbf{B}^a = 0, \tag{1.127}$$

$$\boldsymbol{\nabla} \times \mathbf{B}^a = \mu_0 \mathbf{J}^a, \tag{1.128}$$

and

$$\boldsymbol{\nabla} \cdot \mathbf{J}^a = \boldsymbol{\nabla} \cdot \boldsymbol{\nabla} \times \mathbf{B}^a = 0 \tag{1.129}$$

so it follows from charge current conservation that:

$$\frac{\partial \rho^a}{\partial t} = 0. \tag{1.130}$$

If it is assumed that the scalar potential is zero in magnetostatics, the usual assumption, then:

$$\mathbf{J}^a = \frac{1}{\mu_0} \left(\boldsymbol{\omega}^a{}_b \times \mathbf{B}^b - \mathbf{A}^b \times \mathbf{R}^a{}_b(\text{spin}) \right) \tag{1.131}$$

because there is no electric field present. It follows from Eqs. (1.129) and (1.131) that

$$\boldsymbol{\nabla} \cdot \boldsymbol{\omega}^a{}_b \times \mathbf{B}^b = \boldsymbol{\nabla} \cdot \mathbf{A}^b \times \mathbf{R}^a{}_b(\text{spin}) \tag{1.132}$$

in ECE magnetostatics.

In UFT258 and immediately preceding papers of this series it has been shown that in the absence of a magnetic monopole:

$$\boldsymbol{\omega}^a{}_b \cdot \mathbf{B}^b = \mathbf{A}^b \cdot \mathbf{R}^a{}_b(\text{spin}) \tag{1.133}$$

and that the space part of the Cartan identity in the absence of a magnetic monopole gives the two equations:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b = 0 \tag{1.134}$$

and

$$\boldsymbol{\omega}^a{}_b \cdot \boldsymbol{\nabla} \times \mathbf{A}^b = \mathbf{A}^b \cdot \boldsymbol{\nabla} \times \boldsymbol{\omega}^a{}_b. \tag{1.135}$$

In ECE physics the magnetic flux density is:

$$\mathbf{B}^a = \boldsymbol{\nabla} \times \mathbf{A}^a - \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b \tag{1.136}$$

so the Beltrami equation gives:

$$\boldsymbol{\nabla} \times \mathbf{B}^a = \kappa \mathbf{B}^a = \kappa \left(\boldsymbol{\nabla} \times \mathbf{A}^a - \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b \right). \tag{1.137}$$

Eq. (1.134) from the space part of the Cartan identity is also a Beltrami equation, as is any non-divergent equation:

$$\boldsymbol{\nabla} \times \left(\boldsymbol{\omega}^a{}_b \times \mathbf{A}^b \right) = \kappa \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b. \tag{1.138}$$

From Eq. (1.137):

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{A}^a) - \boldsymbol{\nabla} \times \left(\boldsymbol{\omega}^a{}_b \times \mathbf{A}^b \right) = \kappa \left(\boldsymbol{\nabla} \times \mathbf{A}^a - \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b \right). \quad (1.139)$$

Using Eq. (1.138):

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{A}^a) = \kappa \boldsymbol{\nabla} \times \mathbf{A}^a \quad (1.140)$$

which implies that the vector potential is also defined in general by a Beltrami equation:

$$\boldsymbol{\nabla} \times \mathbf{A}^a = \kappa \mathbf{A}^a \quad (1.141)$$

QED. This is a generally valid result of ECE physics which implies that:

$$\boldsymbol{\nabla} \cdot \mathbf{A}^a = 0. \quad (1.142)$$

From Eq. (1.110) it follows that:

$$\frac{\partial \rho^a}{\partial t} = 0 \quad (1.143)$$

is a general result of ECE physics. From Eqs. (1.135) and (1.141):

$$\boldsymbol{\nabla} \times \boldsymbol{\omega}^a{}_b = \kappa \boldsymbol{\omega}^a{}_b \quad (1.144)$$

so the spin connection vector of ECE physics is also defined in general by a Beltrami equation. This important result can be cross checked for internal consistency using note 258(4) on www.aias.us, starting from Eq. (1.50) of this paper. Considering the X component for example:

$$\omega^a{}_{Xb} \left(\boldsymbol{\nabla} \times \mathbf{A}^a \right)_X = A^b{}_X \left(\boldsymbol{\nabla} \times \boldsymbol{\omega}^a{}_b \right)_X \quad (1.145)$$

and it follows that:

$$\frac{1}{A^{(1)}{}_X} \left(\boldsymbol{\nabla} \times \mathbf{A}^{(1)} \right)_X = \frac{1}{\omega^{(a)}{}_{X(1)}} \left(\boldsymbol{\nabla} \times \boldsymbol{\omega}^a{}_{(1)} \right)_X \quad (1.146)$$

and similarly for the Y and Z components. In order for this to be a Beltrami equation, Eqs. (1.141) and (1.144) must be true, QED.

In magnetostatics there are additional results which emerge as follows. From vector analysis:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\omega}^a{}_b \times \mathbf{B}^b = \mathbf{B}^b \cdot \boldsymbol{\nabla} \times \boldsymbol{\omega}^a{}_b - \boldsymbol{\omega}^a{}_b \cdot \boldsymbol{\nabla} \times \mathbf{B}^b \quad (1.147)$$

and

$$\nabla \cdot \mathbf{A}^b \times \mathbf{R}^a{}_b(\text{spin}) = \mathbf{R}^a{}_b(\text{spin}) \cdot \nabla \times \mathbf{A}^a - \mathbf{A}^b \cdot \nabla \times \mathbf{R}^a{}_b(\text{spin}). \quad (1.148)$$

It is immediately clear that Eqs. (1.87) and (1.144) give Eq. (1.147) self consistently, QED. Eq. (1.148) gives

$$\nabla \cdot \boldsymbol{\omega}^a{}_b \times \mathbf{B}^b = \nabla \cdot \mathbf{A}^b \times \mathbf{R}^a{}_b(\text{spin}) = 0 \quad (1.149)$$

and using Eq. (1.148):

$$\nabla \times \mathbf{R}^a{}_b(\text{spin}) = \kappa \mathbf{R}^a{}_b(\text{spin}) \quad (1.150)$$

so the spin curvature is defined by a Beltrami equation in magnetostatics. Also in magnetostatics:

$$\nabla \times \mathbf{B}^a = \kappa \mathbf{B}^a = \mu_0 \mathbf{J}^a \quad (1.151)$$

so it follows that the current density of magnetostatics is also defined by a Beltrami equation:

$$\nabla \times \mathbf{J}^a = \kappa \ \mathbf{J}^a. \quad (1.152)$$

All these Beltrami equations in general have intricate flow structures graphed following sections of this chapter and animated on www.aias.us. As discussed in Eqs. (1.31) to (1.35) of Note 258(5) on www.aias.us, plane wave structures and O(3) electrodynamics [1-10] are also defined by Beltrami equations. The latter give simple solutions for vacuum plane waves. In other cases the solutions become intricate. The B(3) field is defined by the simplest type of Beltrami equation

$$\nabla \times \mathbf{B}^{(3)} = 0 \ \mathbf{B}^{(3)}. \quad (1.153)$$

In photon mass theory therefore:

$$\nabla \times \mathbf{A}^a = \kappa \ \mathbf{A}^a, \quad (1.154)$$

$$\left(\Box + \left(\frac{mc}{\hbar} \right)^2 \right) \mathbf{A}^a = \mathbf{0}. \quad (1.155)$$

It follows from Eq. (1.154) that:

$$\nabla \cdot \mathbf{A}^a = 0 \quad (1.156)$$

so:

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{A}^a) = \kappa \ \boldsymbol{\nabla} \times \mathbf{A}^a = \kappa^2 \ \mathbf{A}^a \tag{1.157}$$

produces the Helmholtz wave equation:

$$\left(\boldsymbol{\nabla}^2 + \kappa^2\right) \mathbf{A}^a = 0. \tag{1.158}$$

Eq. (1.155) is

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla}^2 + \left(\frac{mc}{\hbar}\right)^2\right) \mathbf{A}^a = \mathbf{0} \tag{1.159}$$

so:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \kappa^2 + \left(\frac{mc}{\hbar}\right)^2\right) \mathbf{A}^a = \mathbf{0}. \tag{1.160}$$

Now use:

$$\mathbf{p} = \hbar \boldsymbol{\kappa} \tag{1.161}$$

and:

$$\frac{\partial^2}{\partial t^2} = -\frac{E^2}{\hbar^2} \tag{1.162}$$

to find that Eq. (1.160) is the Einstein energy equation for the photon of mass m, so the analysis is rigorously self-consistent, QED.

In ECE physics the Lorenz gauge is:

$$\partial_\mu A^{a\mu} = 0 \tag{1.163}$$

i.e.

$$\frac{1}{c^2}\frac{\partial \phi^a}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{A}^a = 0 \tag{1.164}$$

with the solution:

$$\frac{\partial \phi^a}{\partial t} = \boldsymbol{\nabla} \cdot \mathbf{A}^a = 0. \tag{1.165}$$

This is again a general result of ECE physics applicable under any circumstances. Also in ECE physics in general the spin connection vector has no divergence:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\omega}^a{}_b = 0 \tag{1.166}$$

because:

$$\boldsymbol{\nabla} \times \boldsymbol{\omega}^a{}_b = \kappa\ \boldsymbol{\omega}^a{}_b. \tag{1.167}$$

Another rigorous test for self-consistency is given by the definition of the magnetic field in ECE physics:

$$\mathbf{B}^a = \boldsymbol{\nabla} \times \mathbf{A}^a - \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b \tag{1.168}$$

so:

$$\boldsymbol{\nabla} \cdot \mathbf{B}^a = -\boldsymbol{\nabla} \cdot \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b = 0 \tag{1.169}$$

By vector analysis:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b = \mathbf{A}^b \cdot \boldsymbol{\nabla} \times \boldsymbol{\omega}^a{}_b - \boldsymbol{\omega}^a{}_b \cdot \boldsymbol{\nabla} \times \mathbf{A}^b = 0 \tag{1.170}$$

because

$$\boldsymbol{\nabla} \times \boldsymbol{\omega}^a{}_b = \kappa\ \boldsymbol{\omega}^a{}_b, \tag{1.171}$$

$$\boldsymbol{\nabla} \times \mathbf{A}^b = \kappa\ \mathbf{A}^b, \tag{1.172}$$

and:

$$\boldsymbol{\nabla} \cdot \mathbf{A}^b = 0, \tag{1.173}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{\omega}^a{}_b = 0. \tag{1.174}$$

In the absence of a magnetic monopole Eq. (1.84) also follows from the space part of the Cartan identity. So the entire analysis is rigorously self-consistent. The cross consistency of the Beltrami and ECE equations can be checked using:

$$\mathbf{B}^b = \kappa\ \mathbf{A}^b - \boldsymbol{\omega}^b{}_c \times \mathbf{A}^c \tag{1.175}$$

as in note 258(1) on www.aias.us. Eq. (1.175) follows from Eqs. (1.168) and (1.172). Multiply Eq. (1.175) by $\boldsymbol{\omega}^a{}_b$ and use Eq. (1.133) to find:

$$\kappa\ \boldsymbol{\omega}^a{}_b \cdot \mathbf{A}^b - \boldsymbol{\omega}^a{}_b \cdot \boldsymbol{\omega}^b{}_c \times \mathbf{A}^c = \mathbf{A}^b \cdot \mathbf{R}^a{}_b(\text{spin}). \tag{1.176}$$

Now use:

$$\boldsymbol{\omega}^a{}_b \cdot \boldsymbol{\omega}^b{}_c \times \mathbf{A}^c = \mathbf{A}^c \cdot \left(\boldsymbol{\omega}^a{}_b \times \boldsymbol{\omega}^b{}_c\right) \tag{1.177}$$

and relabel summation indices to find:

$$\kappa\ \boldsymbol{\omega}^a{}_b \cdot \mathbf{A}^b - \mathbf{A}^b \cdot (\boldsymbol{\omega}^a{}_c \times \boldsymbol{\omega}^c{}_b) = \mathbf{A}^b \cdot \mathbf{R}^a{}_b(\text{spin}). \tag{1.178}$$

It follows that:

$$\mathbf{R}^a{}_b(\text{spin}) = \kappa\ \boldsymbol{\omega}^a{}_b - \boldsymbol{\omega}^a{}_c \times \boldsymbol{\omega}^c{}_b = \boldsymbol{\nabla} \times \boldsymbol{\omega}^a{}_b - \boldsymbol{\omega}^a{}_c \times \boldsymbol{\omega}^c{}_b \tag{1.179}$$

QED. The analysis correctly and self consistently produces the correct definition of the spin curvature.

Finally, on the U(1) level for the sake of illustration, consider the Beltrami equations of note 258(3) on www.aias.us:

$$\boldsymbol{\nabla} \times \mathbf{A} = \kappa\ \mathbf{A} \tag{1.180}$$

and

$$\boldsymbol{\nabla} \times \mathbf{B} = \kappa\ \mathbf{B} \tag{1.181}$$

in the Ampere Maxwell law

$$\boldsymbol{\nabla} \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}. \tag{1.182}$$

It follows that:

$$\kappa^2\ \mathbf{A} = \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} \tag{1.183}$$

where:

$$\mathbf{E} = -\boldsymbol{\nabla}\phi - \frac{\partial \mathbf{A}}{\partial t}. \tag{1.184}$$

Therefore

$$\kappa^2\ \mathbf{A} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial}{\partial t}\left(-\boldsymbol{\nabla}\phi - \frac{\partial \mathbf{A}}{\partial t}\right) \tag{1.185}$$

and using the Lorenz condition:

$$\boldsymbol{\nabla} \cdot \mathbf{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0 \tag{1.186}$$

it follows that:

$$\frac{\partial \phi}{\partial t} = 0. \tag{1.187}$$

Using

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla}^2 \tag{1.188}$$

Eq. (1.185) becomes the d'Alembert equation in the presence of current density:

$$\Box \mathbf{A} = \mu_0 \mathbf{J}. \tag{1.189}$$

The solutions of the d'Alembert equation (1.189) may be found from:

$$\mathbf{B} = \kappa\ \mathbf{A} \tag{1.190}$$

showing in another way that as soon as the Beltrami equation (1.87) is used, U(1) gauge invariance is refuted.

1.3 Electrostatics, Spin Connection Resonance and Beltrami Structures

As argued already the first Cartan structure equation defines the electric field strength as:

$$\mathbf{E}^a = -c\boldsymbol{\nabla} A^a{}_0 - \frac{\partial \mathbf{A}^a}{\partial t} - c\omega^a{}_{0b}\mathbf{A}^b + cA^b{}_0\boldsymbol{\omega}^a{}_b \tag{1.191}$$

where the four potential of ECE electrodynamics is defined by:

$$A^a{}_\mu = (A^a{}_0, -\mathbf{A}^a) = \left(\frac{\phi^a}{c}, -\mathbf{A}^a\right). \tag{1.192}$$

Here ϕ^a is the scalar potential. If it is assumed that the subject of electrostatics is defined by:

$$\mathbf{B}^a = 0, \quad \mathbf{A}^a = 0, \quad \mathbf{J}^a = 0 \tag{1.193}$$

then the Coulomb law in ECE theory is given by:

$$\boldsymbol{\nabla} \cdot \mathbf{E}^a = \boldsymbol{\omega}^a{}_b \cdot \mathbf{E}^b. \tag{1.194}$$

The electric current in ECE theory is defined by:

$$\mathbf{J}^a = \epsilon_0\, c\left(\omega^a{}_{0b}\mathbf{E}^b - cA^b{}_0\mathbf{R}^a{}_b(\text{orb}) + c\boldsymbol{\omega}^a{}_b \times \mathbf{B}^b - c\mathbf{A}^b \times \mathbf{R}^a{}_b(\text{spin})\right) \tag{1.195}$$

where $\mathbf{R}^a{}_b$(spin) is the spin part of the curvature vector and where $\mathbf{B}^b$ is the magnetic flux density. From Eqs. (1.193) and (1.195):

$$\mathbf{J}^a = \mathbf{0} = \epsilon_0 \ c \left(\omega^a{}_{0b} \mathbf{E}^b - cA^b{}_0 \mathbf{R}^a{}_b(\text{orb}) \right) \tag{1.196}$$

so in ECE electrostatics:

$$\omega^a{}_{0b} \mathbf{E}^b = cA^b{}_0 \mathbf{R}^a{}_b(\text{orb}) \tag{1.197}$$

and

$$\mathbf{E}^a = -c \boldsymbol{\nabla} A^a{}_0 + cA^b{}_0 \boldsymbol{\omega}^a{}_b \tag{1.198}$$

with

$$\boldsymbol{\nabla} \times \mathbf{E}^a = \mathbf{0}. \tag{1.199}$$

From Eqs. (1.198) and (1.199)

$$\boldsymbol{\nabla} \times \mathbf{E}^a = c \boldsymbol{\nabla} \times (A^b{}_0 \boldsymbol{\omega}^a{}_b) \tag{1.200}$$

so we obtain the constraint:

$$\boldsymbol{\nabla} \times (A^b{}_0 \boldsymbol{\omega}^a{}_b) = \mathbf{0}. \tag{1.201}$$

The magnetic charge density in ECE theory is given by:

$$\rho^a_{\text{magn}} = \epsilon_0 \ c \left(\boldsymbol{\omega}^a{}_b \cdot \mathbf{B}^b - \mathbf{A}^b \cdot \mathbf{R}^a{}_b(\text{spin}) \right) \tag{1.202}$$

and the magnetic current density by:

$$\mathbf{J}^a_{\text{magn}} = \epsilon_0 \left(\boldsymbol{\omega}^a{}_b \times \mathbf{E}^b - c\omega^a{}_{0b} \mathbf{B}^b - c \left(\mathbf{A}^b \times \mathbf{R}^a{}_b(\text{orb}) - A^b{}_0 \mathbf{R}^a{}_b(\text{spin}) \right) \right). \tag{1.203}$$

These are thought to vanish experimentally in electromagnetism, so:

$$\boldsymbol{\omega}^a{}_b \cdot \mathbf{B}^b = \mathbf{A}^b \cdot \mathbf{R}^a{}_b(\text{spin}) \tag{1.204}$$

and

$$\boldsymbol{\omega}^a{}_b \times \mathbf{E}^a - c\omega^a{}_{0b} \mathbf{B}^b - c\mathbf{A}^b \times \mathbf{R}^a{}_b(\text{orb}) + cA^b{}_0 \mathbf{R}^a{}_b(\text{spin}) = \mathbf{0}. \tag{1.205}$$

In ECE electrostatics Eq. (1.204) is true automatically because:

$$\mathbf{B}^b = \mathbf{0}, \quad \mathbf{A}^b = \mathbf{0} \tag{1.206}$$

and Eq. (1.203) becomes:

$$\boldsymbol{\omega}^a{}_b \times \mathbf{E}^b + cA^b{}_0\mathbf{R}^a{}_b(\text{spin}) = \mathbf{0}. \tag{1.207}$$

So the equations of ECE electrostatics are:

$$\boldsymbol{\nabla} \cdot \mathbf{E}^a = \boldsymbol{\omega}^a{}_b \cdot \mathbf{E}^b \tag{1.208}$$

$$\omega^a{}_{0b}\mathbf{E}^b = \phi^b\mathbf{R}^a{}_b(\text{orb}) \tag{1.209}$$

$$\boldsymbol{\omega}^a{}_b \times \mathbf{E}^b + \phi^b\mathbf{R}^a{}_b(\text{spin}) = \mathbf{0} \tag{1.210}$$

$$\mathbf{E}^a = -\boldsymbol{\nabla}\phi^a + \phi^b\boldsymbol{\omega}^a{}_b \tag{1.211}$$

Later on in this chapter it is shown that these equations lead to a solution in terms of Bessel functions, but not to Euler Bernoulli resonance.

In order to obtain spin connection resonance Eq. (1.208) must be extended to:

$$\boldsymbol{\nabla} \cdot \mathbf{E}^a = \boldsymbol{\omega}^a{}_b \cdot \mathbf{E}^b - c\mathbf{A}^b(\text{vac}) \cdot \mathbf{R}^a{}_b(\text{orb}) \tag{1.212}$$

where $\mathbf{A}^b(\text{vac})$ is the Eckardt Lindstrom vacuum potential [1-10]. The static electric field is defined by:

$$\mathbf{E}^a = -\boldsymbol{\nabla}\phi^a + \phi^b\boldsymbol{\omega}^a{}_b \tag{1.213}$$

so from Eqs. (1.212) and (1.213):

$$\boldsymbol{\nabla}^2\phi^a + \left(\boldsymbol{\omega}^a{}_b \cdot \boldsymbol{\omega}^b{}_c\right)\phi^c = \boldsymbol{\nabla}\cdot\left(\phi^b\boldsymbol{\omega}^a{}_b\right) + \boldsymbol{\omega}^a{}_b\cdot\boldsymbol{\nabla}\phi^b + c\mathbf{A}^b(\text{vac})\cdot\mathbf{R}^a{}_b(\text{orb}). \tag{1.214}$$

By the ECE antisymmetry law:

$$-\boldsymbol{\nabla}\phi^a = \phi^b\boldsymbol{\omega}^a{}_b \tag{1.215}$$

leading to the Euler Bernoulli resonance equation:

$$\boldsymbol{\nabla}^2\phi^a + \left(\boldsymbol{\omega}^a{}_b \cdot \boldsymbol{\omega}^b{}_c\right)\phi^c = \frac{1}{2}c\mathbf{A}^b(\text{vac}) \cdot \mathbf{R}^a{}_b(\text{orb}) \tag{1.216}$$

and spin connection resonance [1-10]. The left hand side contains the Hooke law term and the right hand side the driving term originating in the vacuum potential. Denote:

$$\rho^a(\text{vac}) = \frac{\epsilon_0\ c}{2}\mathbf{A}^b(\text{vac}) \cdot \mathbf{R}^a{}_b(\text{orb}) \tag{1.217}$$

then the equation becomes:

$$\nabla^2\phi^a + \left(\boldsymbol{\omega}^a{}_b \cdot \boldsymbol{\omega}^b{}_c\right)\phi^c = \frac{\rho^a(\text{vac})}{\epsilon_0}. \tag{1.218}$$

The left hand side of Eq. (1.218) is a field property and the right hand side a property of the ECE vacuum. In the simplest case:

$$\nabla^2\phi + (\omega_0)^2\phi = \frac{\rho(\text{vac})}{\epsilon_0} \tag{1.219}$$

and produces undamped resonance if:

$$\rho(\text{vac}) = \epsilon_0 A\ \cos\omega Z \tag{1.220}$$

where A is a constant. The particular integral of Eq. (1.219) is:

$$\phi = \frac{A\ \cos\omega Z}{(\omega_0)^2 - \omega^2} \tag{1.221}$$

and spin connection resonance occurs at:

$$\omega = \omega_0 \tag{1.222}$$

when:

$$\phi \to \infty \tag{1.223}$$

and there is a resonance peak of electric field strength from the vacuum.

Later in this chapter solutions of Eq. (1.218) are given in terms of a combination of Bessel functions, and also an analysis using the Eckardt Lindstrom vacuum potential as a driving term.

In the absence of a magnetic monopole the Cartan identity is, as argued already:

$$\nabla \cdot \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b = 0 \tag{1.224}$$

which implies:

$$\boldsymbol{\omega}^a{}_b \cdot \nabla \times \mathbf{A}^b = \mathbf{A}^b \cdot \nabla \times \boldsymbol{\omega}^a{}_b. \tag{1.225}$$

A possible solution of this equation is:

$$\boldsymbol{\omega}^a{}_b = \epsilon^a{}_{bc}\boldsymbol{\omega}^c \tag{1.226}$$

leading as argued already to a rigorous justification for O(3) electrodynamics. The Cartan identity (1.224) is itself a Beltrami equation:

$$\nabla \times \left(\boldsymbol{\omega}^a{}_b \times \mathbf{A}^b \right) = \kappa \, \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b. \tag{1.227}$$

From Eqs. (1.226) and (1.227):

$$\nabla \times \left(\mathbf{A}^c \times \mathbf{A}^b \right) = \kappa \, \mathbf{A}^c \times \mathbf{A}^b. \tag{1.228}$$

In the complex circular basis:

$$\mathbf{A}^{(1)} \times \mathbf{A}^{(2)} = i \, A^{(0)} \mathbf{A}^{(3)*} \ \text{ et cyclicum} \tag{1.229}$$

so from Eqs. (1.228) and (1.229):

$$\nabla \times \mathbf{A}^{(i)} = \kappa \, \mathbf{A}^{(i)}, \ \ \mathrm{i} = 1, 2, 3 \tag{1.230}$$

which are Beltrami equations as argued earlier in this chapter.

This result can be obtained self consistently using the Gauss law:

$$\nabla \cdot \mathbf{B}^a = 0 \tag{1.231}$$

which as argued already implies the Beltrami equation:

$$\nabla \times \mathbf{B}^a = \kappa \, \mathbf{B}^a. \tag{1.232}$$

From Eqs. (1.168) and (1.232):

$$\nabla \times \mathbf{B}^a = \kappa \, \mathbf{B}^a = \kappa \left(\nabla \times \mathbf{A}^a - \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b \right) \tag{1.233}$$

so:

$$\nabla \times (\nabla \times \mathbf{A}^a) - \nabla \times \left(\boldsymbol{\omega}^a{}_b \times \mathbf{A}^b \right) = \kappa \left(\nabla \times \mathbf{A}^a - \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b \right) \tag{1.234}$$

Using Eq. (1.227) gives:

$$\nabla \times (\nabla \times \mathbf{A}^a) = \kappa \, \nabla \times \mathbf{A}^a \tag{1.235}$$

which implies Eqs. (1.228) to (1.230) QED. As shown earlier in this chapter the Beltrami structure also governs the spin connection vector:

$$\nabla \times \boldsymbol{\omega}^a{}_b = \kappa \, \boldsymbol{\omega}^a{}_b. \tag{1.236}$$

It follows that the equations:

$$\boldsymbol{\omega}^{(3)} = \frac{1}{2}\frac{\kappa}{A^{(0)}}\mathbf{A}^{(3)} \tag{1.237}$$

and:

$$\boldsymbol{\omega}^{(2)} = \frac{1}{2}\frac{\kappa}{A^{(0)}}\mathbf{A}^{(2)} \tag{1.238}$$

produce O(3) electrodynamics [1-10]:

$$\mathbf{B}^{(1)*} = \boldsymbol{\nabla} \times \mathbf{A}^{(1)*} - i\frac{\kappa}{A^{(0)}}\mathbf{A}^{(2)} \times \mathbf{A}^{(3)} \text{ et cyclicum.} \tag{1.239}$$

As shown in Note 259(3) on www.aias.us there are many inter-related equations of O(3) electrodynamics which all originate in geometry.

Later in this chapter it is argued a consequence of these conclusions is that the spin connection and orbital curvature vectors also obey a Beltrami structure.

The fact that ECE is a unified field theory also allows the development and interrelation of several basic equations, including the definition of B(3):

$$\mathbf{B}^{(3)} = \boldsymbol{\nabla} \times \mathbf{A}^{(3)} - i\frac{\kappa}{A^{(0)}}\mathbf{A}^{(1)} \times \mathbf{A}^{(2)}. \tag{1.240}$$

It can be written as:

$$\mathbf{B} = -i\frac{e}{\hbar}\mathbf{A} \times \mathbf{A}^{*} = B^{(0)}\mathbf{k} = B_Z\mathbf{k}. \tag{1.241}$$

Although B(3) is a radiated and propagating field as is well-known [1-10] Eq. (1.241) can be used as a general definition of the magnetic flux density for a choice of potentials. This is important for the subject of magnetostatics and the development [1-10] of the fermion equation with:

$$\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r}. \tag{1.242}$$

Eq. (1.241) gives the transition from classical to quantum mechanics. In ECE electrodynamics **A** must always be a Beltrami field and this is the result of the Cartan identity as already argued. So it is necessary to solve the following equations simultaneously:

$$\mathbf{B} = -i\frac{e}{\hbar}\mathbf{A} \times \mathbf{A}^{*}, \;\; \mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r}, \;\; \boldsymbol{\nabla} \times \mathbf{A} = \kappa\, \mathbf{A}. \tag{1.243}$$

This can be done using the principles of general relativity, so that the electromagnetic field is a rotating and translating frame of reference. The position vector is therefore:

$$\mathbf{r} = \mathbf{r}^* = \frac{r^{(0)}}{\sqrt{2}} (\mathbf{i} - i\mathbf{j})\, e^{i\phi} \tag{1.244}$$

where:

$$\mathbf{r} = \mathbf{r}^{(1)}, \;\; \mathbf{r}^* = \mathbf{r}^{(2)}, \;\; \phi = \omega t - \kappa Z \tag{1.245}$$

so:

$$\mathbf{r}^{(1)} \times \mathbf{r}^{(2)} = i r^{(0)} \mathbf{r}^{(3)*} \;\text{ et cyclicum.} \tag{1.246}$$

It follows that:

$$\boldsymbol{\nabla} \times \mathbf{r}^{(1)} = \kappa\, \mathbf{r}^{(1)} \tag{1.247}$$

$$\boldsymbol{\nabla} \times \mathbf{r}^{(2)} = \kappa\, \mathbf{r}^{(2)} \tag{1.248}$$

$$\boldsymbol{\nabla} \times \mathbf{r}^{(3)} = 0\, \mathbf{r}^{(3)} \tag{1.249}$$

The results (1.246) for plane waves can be generalized to any Beltrami solutions, so it follows that spacetime itself has a Beltrami structure. From Eqs. (1.242) and (1.244):

$$\mathbf{A} = \mathbf{A}^{(1)} = \frac{B^{(0)} r^{(0)}}{2\sqrt{2}} (i\mathbf{i} + \mathbf{j})\, e^{i\phi} = \frac{A^{(0)}}{\sqrt{2}} (i\mathbf{i} + \mathbf{j})\, e^{i\phi} \tag{1.250}$$

where:

$$A^{(0)} = \frac{1}{2} B^{(0)} r^{(0)} \tag{1.251}$$

and from Eq. (1.250):

$$\boldsymbol{\nabla} \times \mathbf{A} = \kappa\, \mathbf{A} \tag{1.252}$$

QED. Therefore it is always possible to write the vector potential in the form (1.242) provided that spacetime itself has a Beltrami structure. This conclusion ties together several branches of physics because Eq. (1.242) is used to produce the Landé factor, ESR, NMR and so on from the Dirac equation, which becomes the fermion equation [1-10] in ECE physics.

As argued already the tetrad postulate and ECE postulate give:

$$\left(\Box + \kappa_0^2\right) \mathbf{A} = \mathbf{0} \tag{1.253}$$

and the fermion or chiral Dirac equation is a factorization of Eq. (1.253). As shown in Chapter 1:

$$\kappa_0^2 = q^{\nu}{}_{a}\partial^{\mu}\left(\omega^{a}{}_{\mu\nu} - \Gamma^{a}{}_{\mu\nu}\right) \tag{1.254}$$

where $q^{\nu}{}_{a}$ is the inverse tetrad, defined by:

$$q^{a}{}_{\nu}q^{\nu}{}_{a} = 1. \tag{1.255}$$

In generally covariant format Eq. (1.253) is:

$$\left(\Box + \kappa_0^2\right) A^{a}{}_{\mu} = 0 \tag{1.256}$$

and with:

$$A^{a}{}_{\mu} = (A^{a}{}_{0}, -\mathbf{A}^{a}) \tag{1.257}$$

it follows that:

$$\left(\Box + \kappa_0^2\right) A_0 = 0, \tag{1.258}$$

$$\left(\Box + \kappa_0^2\right) \mathbf{A} = \mathbf{0}, \tag{1.259}$$

which gives Eq. (1.254) QED. The d'Alembertian is defined by:

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla}^2. \tag{1.260}$$

The Beltrami condition:

$$\boldsymbol{\nabla}\mathbf{A} = \kappa\ \mathbf{A} \tag{1.261}$$

gives the Helmholtz wave equation:

$$\left(\boldsymbol{\nabla}^2 + \kappa^2\right) \mathbf{A} = \mathbf{0} \tag{1.262}$$

if:

$$\boldsymbol{\nabla} \cdot \mathbf{A} = 0. \tag{1.263}$$

From Eq. (1.259):

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla}^2 + \kappa_0^2\right) \mathbf{A} = \mathbf{0} \tag{1.264}$$

so:

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} + \left(\kappa_0^2 + \kappa^2\right)\mathbf{A} = \mathbf{0} \tag{1.265}$$

which is the equation for the time dependence of **A**. The Helmholtz and Beltrami equations are for the space dependence of **A**. Eq. (1.267) is satisfied by:

$$\mathbf{A} = \mathbf{A}_0 \exp(i\omega t) \tag{1.266}$$

where:

$$\frac{\omega^2}{c^2} = \kappa^2 + \kappa_0^2. \tag{1.267}$$

Eq. (1.267) is a generalization of the Einstein energy equation for a free particle:

$$E^2 = c^2p^2 + m^2c^4 \tag{1.268}$$

where:

$$E = \hbar\omega, \quad \mathbf{p} = \hbar\boldsymbol{\kappa} \tag{1.269}$$

using:

$$\kappa_0^2 = \left(\frac{mc}{\hbar}\right)^2 = q^{\nu}{}_a \partial^{\mu}\left(\omega^a{}_{\mu\nu} - \Gamma^a{}_{\mu\nu}\right). \tag{1.270}$$

So mass in ECE theory is defined by geometry.

The general solution of Eq (1.256) is therefore:

$$A^a{}_{\mu} = A^a{}_{\mu}(0)\exp\left(i(\omega t - \kappa Z)\right) \tag{1.271}$$

where:

$$\omega^2 = c^2\left(\kappa^2 + \kappa_0^2\right). \tag{1.272}$$

It follows that there exist the equations:

$$\left(\Box + \kappa_0^2\right)\phi^a = 0 \tag{1.273}$$

and

$$\left(\boldsymbol{\nabla}^2 + \kappa^2\right)\phi^a = 0 \tag{1.274}$$

where ϕ^a is the scalar potential in ECE physics. For each a:

$$\left(\boldsymbol{\nabla}^2 + \kappa^2\right)\phi = 0. \tag{1.275}$$

Now write:

$$\kappa_0 = \frac{mc}{\hbar} \tag{1.276}$$

where m is mass. The relativistic wave equation for each a is:

$$\left(\Box + \kappa_0^2\right)\phi = 0 \tag{1.277}$$

which is the quantized format of:

$$E^2 = c^2p^2 + m^2c^4 = c^2p^2 + \hbar^2\kappa_0^2c^2. \tag{1.278}$$

Eq. (1.278) is:

$$E = \gamma mc^2 \tag{1.279}$$

where the Lorentz factor is:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \tag{1.280}$$

and where the relativistic momentum is:

$$\mathbf{p} = \gamma m\mathbf{v}. \tag{1.281}$$

Define the relativistic energy as:

$$T = E - mc^2 \tag{1.282}$$

and it follows that:

$$T = (\gamma - 1)mc^2 \underset{v \ll c}{\longrightarrow} \frac{1}{2}mv^2 \tag{1.283}$$

which is the non-relativistic limit of the kinetic energy, i.e.:

$$T = \frac{p^2}{2m}. \tag{1.284}$$

Using:

$$T = i\hbar\frac{\partial}{\partial t}, \quad \mathbf{p} = -i\hbar\boldsymbol{\nabla} \tag{1.285}$$

Eq. (1.284) quantizes to the free particle Schroedinger equation:

$$-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\phi = T\phi \tag{1.286}$$

which is the Helmholtz equation:

$$\left(\boldsymbol{\nabla}^2 + \frac{2mT}{\hbar^2}\right)\phi = 0. \tag{1.287}$$

It follows that the free particle Schroedinger equation is a Beltrami equation but with the vector potential replaced by the scalar potential ϕ, which plays the role of the wavefunction. It also follows in the non-relativistic limit that:

$$\left(\boldsymbol{\nabla}^2 + \frac{2mT}{\hbar^2}\right)\mathbf{A} = \mathbf{0}, \tag{1.288}$$

so:

$$\kappa^2 = \frac{2mT}{\hbar^2}. \tag{1.289}$$

The Helmholtz equation (1.287) can be written as:

$$\left(\boldsymbol{\nabla}^2 + \kappa^2\right)\phi = 0 \tag{1.290}$$

which is an Euler Bernoulli equation without a driving term on the right hand side. In the presence of potential energy V Eq. (1.286) becomes:

$$H\phi = \left(-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + V\right)\phi = E\phi \tag{1.291}$$

where H is the Hamiltonian and E the total energy:

$$E = T + V \tag{1.292}$$

Eq. (1.291) is:

$$\left(\boldsymbol{\nabla}^2 + \kappa^2\right)\phi = \frac{2mV}{\hbar^2}\phi \tag{1.293}$$

which is an inhomogeneous Helmholtz equation similar to an Euler Bernoulli resonance equation with a driving term on the right hand side. However Eq. (1.293) is an eigenequation rather than an Euler Bernoulli equation as conventionally defined, but Eq. (1.293) has very well-known resonance solutions in quantum mechanics. Eq. (1.293) may be written as:

$$\left(\boldsymbol{\nabla}^2 + \kappa_1^2\right)\phi = 0 \tag{1.294}$$

where:

$$\kappa_1^2 = \frac{2m}{\hbar^2}(E - V) \tag{1.295}$$

and in UFT226 ff. on www.aias.us was used in the theory of low energy nuclear reactions (LENR). Eq. (1.294) is well known to be a linear oscillator equation which can be used to define the structure of the atom and nucleus. It can be transformed into an Euler Bernoulli equation as follows:

$$\left(\boldsymbol{\nabla}^2 + \kappa_1^2\right)\phi = A\cos(\kappa_2 Z) \tag{1.296}$$

where the right hand side represents a vacuum potential. It is exactly the structure obtained from the ECE Coulomb law as argued already.

1.4 The Beltrami Equation for Linear Momentum

The free particle Schroedinger equation can be obtained from the Beltrami equation for momentum:

$$\boldsymbol{\nabla} \times \mathbf{p} = \kappa \mathbf{p} \tag{1.297}$$

which can be developed into the Helmholtz equation:

$$\left(\boldsymbol{\nabla}^2 + \kappa^2\right)\mathbf{p} = \mathbf{0} \tag{1.298}$$

if it is assumed that:

$$\boldsymbol{\nabla} \cdot \mathbf{p} = 0. \tag{1.299}$$

If $\mathbf{p}$ is a linear momentum in the classical straight line then:

$$\kappa = 0. \tag{1.300}$$

In general however $\mathbf{p}$ has intricate Beltrami solutions, some of which are animated in UFT258 on www.aias.us and its animation section.

Now quantize Eq. (1.298):

$$\mathbf{p}\psi = -i\hbar\boldsymbol{\nabla}\psi \tag{1.301}$$

so:

$$\left(\boldsymbol{\nabla}^2 + \kappa^2\right)\boldsymbol{\nabla}\psi = \mathbf{0}. \tag{1.302}$$

Use:

$$\boldsymbol{\nabla}^2\boldsymbol{\nabla}\psi = \boldsymbol{\nabla}\boldsymbol{\nabla}^2\psi \tag{1.303}$$

and:

$$\boldsymbol{\nabla}(\kappa^2\psi) = \kappa^2\boldsymbol{\nabla}\psi \tag{1.304}$$

assuming that:

$$\boldsymbol{\nabla}\kappa = \mathbf{0} \tag{1.305}$$

to arrive at:

$$\boldsymbol{\nabla}\left(\left(\boldsymbol{\nabla}^2 + \kappa^2\right)\psi\right) = \mathbf{0}. \tag{1.306}$$

A possible solution is:

$$\left(\boldsymbol{\nabla}^2 + \kappa^2\right)\psi = \mathbf{0} \tag{1.307}$$

which is the Helmholtz equation for the scalar ψ, the wave function of quantum mechanics. The Schroedinger equation for a free particle is obtained by applying Eq. (1.301) to:

$$E = \frac{p^2}{2m} \tag{1.308}$$

so:

$$-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi = E\psi \tag{1.309}$$

and:

$$\left(\boldsymbol{\nabla}^2 + \frac{2Em}{\hbar^2}\right)\psi = 0. \tag{1.310}$$

Eqs. (1.307) and (1.310) are the same if:

$$\kappa^2 = \frac{2Em}{\hbar^2} \tag{1.311}$$

QED. Using the de Broglie relation:

$$\mathbf{p} = \hbar\boldsymbol{\kappa} \tag{1.312}$$

then:

$$p^2 = 2Em \tag{1.313}$$

which is Eq. (1.308), QED. Therefore the free particle Schroedinger equation is the Beltrami equation:

$$\boldsymbol{\nabla} \times \mathbf{p} = \left(\frac{2Em}{\hbar^2}\right)^{1/2} \mathbf{p} \tag{1.314}$$

with:

$$\mathbf{p}\psi = -i\hbar\boldsymbol{\nabla}\psi. \tag{1.315}$$

The free particle Schroedinger equation originates in the Beltrami equation.

This method can be extended to the general Schroedinger equation in which the potential energy V is present. Consider the momentum Beltrami equation (1.297) in the general case where κ depends on coordinates. Taking the curl of both sides of Eq. (1.297):

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{p}) = \boldsymbol{\nabla} \times (\kappa\ \mathbf{p})\,. \tag{1.316}$$

By vector analysis Eq. (1.316) can be developed as:

$$\boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{p}) - \boldsymbol{\nabla}^2\mathbf{p} = \kappa^2\mathbf{p} + \boldsymbol{\nabla}\kappa \times \mathbf{p} \tag{1.317}$$

so:

$$\left(\boldsymbol{\nabla}^2 + \kappa^2\right)\mathbf{p} = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{p}) - \boldsymbol{\nabla}\kappa \times \mathbf{p}. \tag{1.318}$$

One possible solution is:

$$\left(\boldsymbol{\nabla}^2 + \kappa^2\right)\mathbf{p} = \mathbf{0} \tag{1.319}$$

and

$$\boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{p}) = \boldsymbol{\nabla}\kappa \times \mathbf{p}. \tag{1.320}$$

Eq. (1.320) implies

$$\mathbf{p} \cdot \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{p}) = \mathbf{p} \cdot \boldsymbol{\nabla}\kappa \times \mathbf{p} = 0. \tag{1.321}$$

Two possible solutions of Eq. (1.321) are:

$$\boldsymbol{\nabla} \cdot \mathbf{p} = 0 \tag{1.322}$$

and

$$\boldsymbol{\nabla}\left(\boldsymbol{\nabla}\cdot\mathbf{p}\right)=\mathbf{0}. \tag{1.323}$$

Using the quantum postulate (1.301) in Eq. (1.319) gives:

$$\left(\boldsymbol{\nabla}^2+\kappa^2\right)\boldsymbol{\nabla}\psi=\mathbf{0} \tag{1.324}$$

and the Schroedinger equation [1-10]:

$$\left(\boldsymbol{\nabla}^2+\kappa^2\right)\psi=0. \tag{1.325}$$

From Eq. (1.325)

$$\boldsymbol{\nabla}\left(\left(\boldsymbol{\nabla}^2+\kappa^2\right)\psi\right)=\mathbf{0} \tag{1.326}$$

i. e.

$$\left(\boldsymbol{\nabla}^2+\kappa^2\right)\boldsymbol{\nabla}\psi+\left(\boldsymbol{\nabla}\left(\boldsymbol{\nabla}^2+\kappa^2\right)\right)\psi=\mathbf{0}, \tag{1.327}$$

a possible solution of which is:

$$\left(\boldsymbol{\nabla}^2+\kappa^2\right)\boldsymbol{\nabla}\psi=\mathbf{0} \tag{1.328}$$

and

$$\left(\boldsymbol{\nabla}\left(\boldsymbol{\nabla}^2+\kappa^2\right)\right)\psi=\mathbf{0}. \tag{1.329}$$

Eq. (1.329) is Eq. (1.324), QED. Eq. (1.329) can be written as:

$$\boldsymbol{\nabla}\boldsymbol{\nabla}^2\psi+\boldsymbol{\nabla}\kappa^2\psi=\mathbf{0} \tag{1.330}$$

i. e.

$$\boldsymbol{\nabla}\left(\boldsymbol{\nabla}^2\psi+\kappa^2\psi\right)=\mathbf{0}. \tag{1.331}$$

A possible solution of Eq. (1.331) is the Schroedinger equation:

$$\left(\boldsymbol{\nabla}^2+\kappa^2\right)\psi=0. \tag{1.332}$$

So the Schroedinger equation is compatible with Eq. (1.324).

Eq. (1.322) gives:

$$\boldsymbol{\nabla}^2\psi=0 \tag{1.333}$$

which is consistent with Eq. (1.332) only if:

$$\kappa^2 = 0. \tag{1.334}$$

Eq. (1.323) gives:

$$\boldsymbol{\nabla}\left(\boldsymbol{\nabla}^2\psi\right) = \mathbf{0} \tag{1.335}$$

where:

$$\boldsymbol{\nabla}^2\psi = -\kappa^2\psi. \tag{1.336}$$

Therefore:

$$\boldsymbol{\nabla}\left(\kappa^2\psi\right) = \left(\boldsymbol{\nabla}\kappa^2\right)\psi + \kappa^2\boldsymbol{\nabla}\psi \tag{1.337}$$

and:

$$\boldsymbol{\nabla}\psi = -\left(\frac{\boldsymbol{\nabla}\kappa^2}{\kappa^2}\right)\psi. \tag{1.338}$$

Therefore:

$$\begin{aligned}\boldsymbol{\nabla}\cdot\boldsymbol{\nabla}\psi = \boldsymbol{\nabla}^2\psi &= -\boldsymbol{\nabla}\cdot\left(\frac{\boldsymbol{\nabla}\kappa^2}{\kappa^2}\psi\right) \\ &= -\left(\boldsymbol{\nabla}\cdot\left(\frac{\boldsymbol{\nabla}\kappa^2}{\kappa^2}\right)\right)\psi - \left(\frac{\boldsymbol{\nabla}\kappa^2}{\kappa^2}\right)\boldsymbol{\nabla}\psi.\end{aligned} \tag{1.339}$$

From a comparison of Eqs. (1.332) and (1.339) we obtain the subsidiary condition:

$$\boldsymbol{\nabla}^2\kappa^2 = \kappa^4 \tag{1.340}$$

where:

$$\kappa^2 = \frac{2m}{\hbar^2}\left(V - E\right). \tag{1.341}$$

Therefore:

$$\boldsymbol{\nabla}\kappa^2 = \frac{2m}{\hbar^2}\boldsymbol{\nabla}V \tag{1.342}$$

and

$$\boldsymbol{\nabla}^2\kappa^2 = \frac{2m}{\hbar^2}\boldsymbol{\nabla}^2V \tag{1.343}$$

giving a quadratic constraint in $V - E$:

$$\boldsymbol{\nabla}^2(V-E) = \frac{2m}{\hbar^2}(V-E)^2. \tag{1.344}$$

This can be written as a quadratic equation in E, which is a constant. E is expressed in terms of V, $\boldsymbol{\nabla}V$, and $\boldsymbol{\nabla}^2 V$. Using:

$$\boldsymbol{\nabla}E = \mathbf{0} \tag{1.345}$$

gives a differential equation in V which can be solved numerically, giving an expression for V. Finally this expression for V is used in the Schroedinger equation:

$$\left(-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + V\right)\psi = E\psi \tag{1.346}$$

to find the energy levels of E and the wavefunctions ψ. These are energy levels and wavefunctions of the interior parton structure of an elementary particle such as an electron, proton or neutron. The well-developed methods of computational quantum mechanics can be used to find the expectation values of any property and can be applied to scattering theory, notably deep inelastic electron-electron, electron-proton and electron-neutron scattering. The data are claimed conventionally to provide evidence for quark structure, but the quark model depends on the validity of the U(1) and electroweak sectors of the standard model. In this book these sector theories are refuted in many ways.

1.5 Examples for Beltrami functions

In this section we give some examples of Beltrami fields with the corresponding graphs. We start the demonstration with a general consideration. Marsh [2] defines a general Beltrami field with cylindrical geometry by

$$\mathbf{B} = \begin{bmatrix} 0 \\ B_\theta(r) \\ B_Z(r) \end{bmatrix} \tag{1.347}$$

with cylindrical coordinates r, θ, Z. There is only an r dependence of the field components. For this to be a Beltrami field, the Beltrami condition in cylindrical coordinates

$$\boldsymbol{\nabla}\times\mathbf{B} = \begin{bmatrix} \frac{1}{r}\frac{\partial B_Z}{\partial\theta} - \frac{\partial B_\theta}{\partial Z} \\ \frac{\partial B_r}{\partial Z} - \frac{\partial B_Z}{\partial r} \\ \frac{1}{r}\left(\frac{\partial(r\,B_\theta)}{\partial r} - \frac{\partial B_r}{\partial\theta}\right) \end{bmatrix} = \kappa\,\mathbf{B} \tag{1.348}$$

must hold. The divergence in cylindridal coordinates is

$$\nabla \cdot \mathbf{B} = \frac{1}{r}\frac{\partial(r\,B_r)}{\partial r} + \frac{1}{r}\frac{\partial B_\theta}{\partial \theta} + \frac{\partial B_Z}{\partial Z}. \tag{1.349}$$

Obviously the field (1.347) is divergence-free, a prerequisite to be a Beltrami field. Eq.(1.348) simplifies to

$$\nabla \times \mathbf{B} = \begin{bmatrix} 0 \\ -\frac{\partial B_Z}{\partial r} \\ \frac{\partial B_\theta}{\partial r} + \frac{1}{r}B_\theta. \end{bmatrix} = \kappa \begin{bmatrix} 0 \\ B_\theta \\ B_Z \end{bmatrix}. \tag{1.350}$$

κ can be a function in general. Here we consider the case of constant κ. From the second component of Eq.(1.350) follows

$$-\frac{\partial}{\partial r}\,B_Z = \kappa\,B_\theta \tag{1.351}$$

and from the third component

$$r\,\frac{\partial}{\partial r}\,B_\theta + B_\theta = \kappa\,r\,B_Z. \tag{1.352}$$

Integrating Eq.(1.351), inserting the result for B_Z into (1.352) gives

$$\frac{\partial}{\partial r}\,B_\theta + \frac{B_\theta}{r} = -\kappa^2 \int B_\theta\,dr, \tag{1.353}$$

and differentiating this equation leads to the second order differential equation

$$r^2\,\frac{\partial^2}{\partial r^2}\,B_\theta + r\,\frac{\partial}{\partial r}\,B_\theta + \kappa^2\,r^2\,B_\theta - B_\theta = 0. \tag{1.354}$$

Finally we change the variable r to κr which leads to Bessel's differential equation

$$r^2\,\frac{d^2}{d\,r^2}\,B_\theta\left(\kappa\,r\right) + r\,\frac{d}{d\,r}\,B_\theta\left(\kappa\,r\right) + \left(\kappa^2\,r^2 - 1\right)\,B_\theta\left(\kappa\,r\right) = 0. \tag{1.355}$$

The solution is the Bessel function

$$B_\theta(r) = B_0\,J_1(\kappa r) \tag{1.356}$$

(with a constant B_0) and from (1.351) follows

$$B_Z(r) = B_0\,J_0(\kappa r). \tag{1.357}$$

This is the known solution of Reed/Marsh, scaled by the wave number κ, with longitudinal components. This solution is graphed in Fig. 1.1. The stream lines are shown in Fig. 1.2. It has to be taken in mind that stream lines show how a test particle moves in the vector field which is considered a velocity field:

$$\mathbf{x} + \Delta\mathbf{x} = \mathbf{x} + \mathbf{v}(\mathbf{x})\ \Delta t. \tag{1.358}$$

All streamline examples are started with 9 points in parallel on the X axis so all animations should be comparable.

The general Beltrami field can be written as

$$\mathbf{v} = \kappa\ \boldsymbol{\nabla} \times (\psi\mathbf{a}) + \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times (\psi\mathbf{a}) \tag{1.359}$$

where ψ is an arbitrary function, κ is a constant and $\mathbf{a}$ is a constant vector. In Fig. 1.3 we show an example with

$$\psi = \frac{1}{L^3} XYZ, \tag{1.360}$$

$$\mathbf{a} = [0, 0, 1]. \tag{1.361}$$

The field is coplanar to the XY plane and gives planar streamlines of hyperbolic form.

Another known solution based on Bessel functions is the Lundquist solution

$$\mathbf{v} = \begin{bmatrix} J_1(\kappa r)\lambda e^{-\lambda Z} \\ J_1(\kappa r)\alpha e^{-\lambda Z} \\ J_1(\kappa r) e^{-\lambda Z} \end{bmatrix} \tag{1.362}$$

with

$$\kappa = \sqrt{\alpha^2 + \lambda^2} \tag{1.363}$$

and constants α and λ. The Lundquist function (for $Z > 0$) is graphed in Fig. 1.4 and initially behaves similar to the Bessel case discussed above. However the field shrinks with Z due to the exponential factor. Fig. 1.5 shows a projection into the XY plane. The vectors are always rotated by 45° against the radial direction. Longitudinal parts are not visible here as discussed for the Rodriguez-Vaz case. Outer streamlines (Fig. 1.6) go down to the region $Z < 0$, and here the exponential factor $\exp(-\lambda Z)$ gives an exponential growth, this is well recognizable in the second version of this animation on www.aias.us. λ can be assumed complex-valued as discussed by Reed, leading to oscillatory solutions, but then problems can arise in other parts of the field definition.

Finally we give some graphic examples for plane waves. Although these are well known, it is useful to recall certain features that not always are considered where plane waves are used. In ECE theory their most prominent appearance is in the vector potential of the free electromagnetic field, in cyclic cartesian coordinates:

$$\mathbf{A}_1 = \frac{A_0}{\sqrt{2}} \begin{bmatrix} e^{i\,(\omega\, t - \kappa\, Z)} \\ -i\; e^{i\,(\omega\, t - \kappa\, Z)} \\ 0 \end{bmatrix}, \; \mathbf{A}_2 = \frac{A_0}{\sqrt{2}} \begin{bmatrix} e^{i\,(\omega\, t - \kappa\, Z)} \\ i\; e^{i\,(\omega\, t - \kappa\, Z)} \\ 0 \end{bmatrix}, \; \mathbf{A}_3 = 0. \tag{1.364}$$

Their divergence is zero and the eigenvalue of the curl operator is κ or $-\kappa$, respectively. The plane wave can also be defined as real valued:

$$\mathbf{A}_1 = \frac{A_0}{\sqrt{2}} \begin{bmatrix} \cos(\omega\, t - \kappa\, Z) \\ -\sin(\omega\, t - \kappa\, Z) \\ 0 \end{bmatrix}, \; \mathbf{A}_2 = \frac{A_0}{\sqrt{2}} \begin{bmatrix} \sin(\omega\, t - \kappa\, Z) \\ \cos(\omega\, t - \kappa\, Z) \\ 0 \end{bmatrix}, \; \mathbf{A}_3 = 0 \tag{1.365}$$

and are Beltrami fields also, however with positive eigenvalues for $\mathbf{A}_1$ and $\mathbf{A}_2$. The real-valued plane waves are graphed as vector fields in Fig. 1.7 for a fixed instant of time $t = 0$. $\mathbf{A}_1$ and $\mathbf{A}_2$ are perpendicular to one another and define a rotating frame in Z direction. The streamlines in one plane are all parallel straight lines. To show a variation, they have been graphed in Fig. 1.8 for different starting points on the Z axis. Here the rotation of frames can be seen again.

Streamlines of plane waves are not very instructive concerning the physical meaning of these waves. It is more illustrative to show their time behaviour. We started with streamlines in the XY plane and computed their time evolution. The streamlines would remain in that plane so we added a Z component $v\, t$ to simulate a propagation in that direction as is the case for electromagnetic waves with $v = c$. Thus in Fig. 1.9 the trace of circularly polarized waves is obtained. Interestingly the waves are phase-shifted, although all starting points are at $Y = 0$.

In this paper we are considering plane wave in the context of Beltrami fields. As worked out the fields E, B and A are parallel. Therefore the components $\mathbf{A}_1$ and $\mathbf{A}_2$ do not demonstrate the behaviour of electric and magnetic fields of ordinary transversal electromagnetic fields which are phase-shifted by 90°. Reed [**?**] gives a very good explanation of this extraordinary case:

Every plane wave solution corresponds to two circularly polarized waves propagating oppositely to each other and combining to form a standing wave. This standing wave does not possess the standard power flow feature of linearly- or circularly-polarized waves with $E \perp B$, since the combined Poynting vectors of the circularly-polarized waves cancel each other similar to the situation we met earlier in connection with Beltrami plasma vortex filaments. Essentially, the combination of these two waves produces a standing wave propagating non-zero magnetic helicity. In the book by Marsh [?] the relationship is shown between the helicity and energy densities for this wave as well, as the very interesting fact that any magnetostatic solution to the FFMF equations can be used to construct a solution to Maxwell´s equations with $E \| B$.

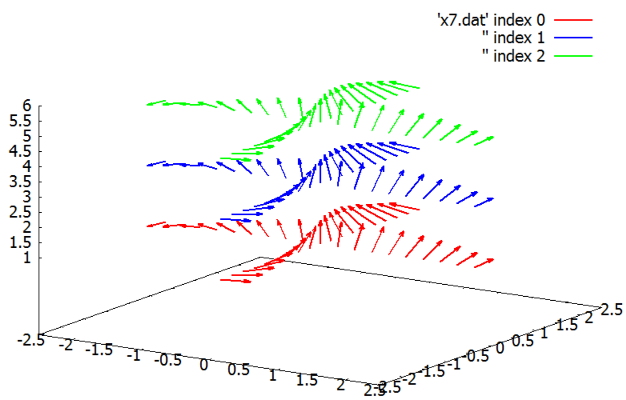

Figure 1.1: Bessel function solution.

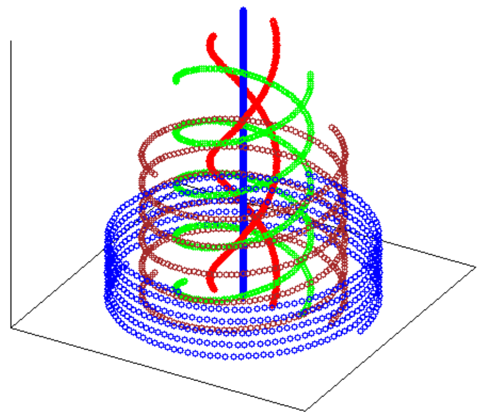

Figure 1.2: Streamlines of the Bessel function solution.

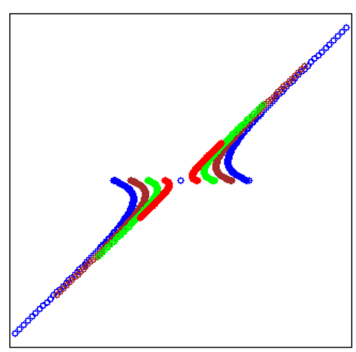

Figure 1.3: General solution with $\psi = \frac{1}{L^3}XYZ$.

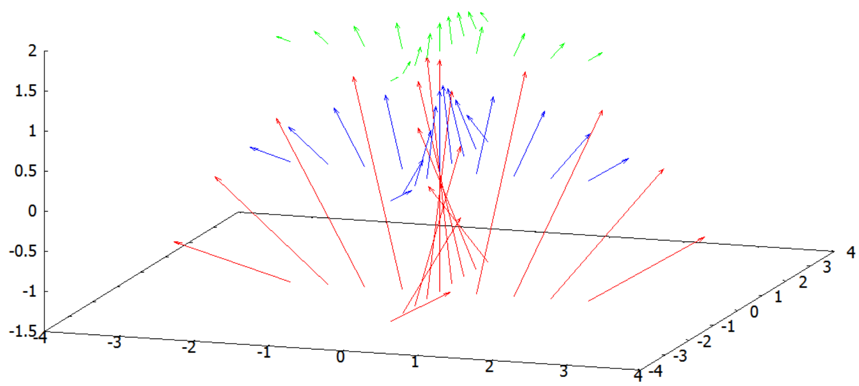

Figure 1.4: Lundquist solution.

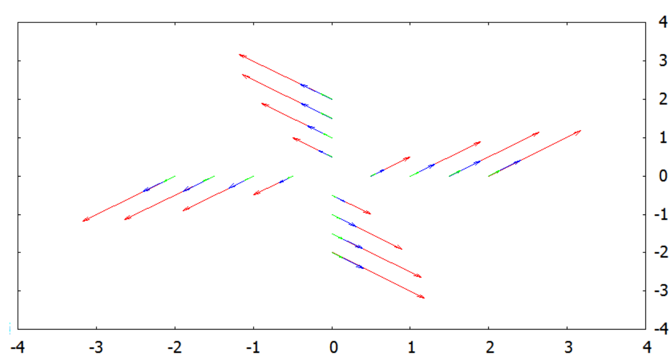

Figure 1.5: Lundquist solution, projected to XY plane.

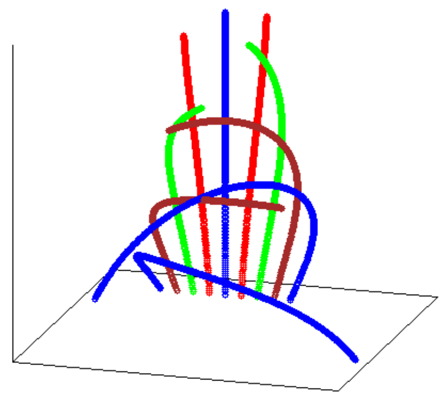

Figure 1.6: Streamlines of Lundquist solution.

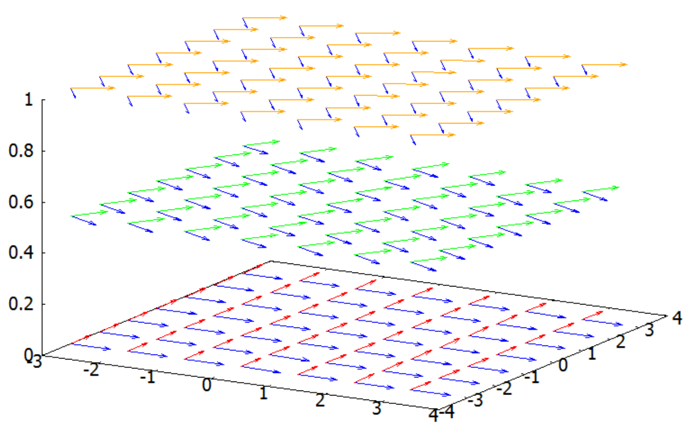

Figure 1.7: Plane wave field, $\mathbf{A}_1$ and $\mathbf{A}_2$.

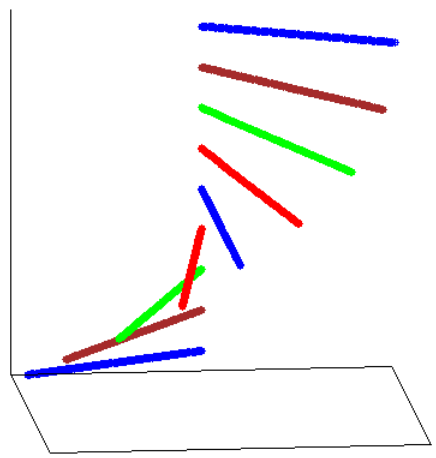

Figure 1.8: Streamlines of plane waves.

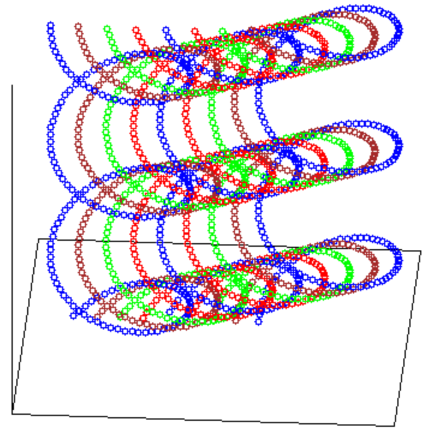

Figure 1.9: Time evolution of points transported by plane waves.

1.6 Parton Structure of Elementary Particles

We develop a solution of the constraint Schroedinger equation (**??**) on basis of the Beltrami equations developed in this chapter. The solution is applied to elementary particles and reveals their so-called Parton structure.

Solution of the constraint equation (1.340)

Before solving the Schroedinger equation (346), the potential is derived from the constraint equation (1.340) or (1.344), respectively. We choose the form (1.340) for κ^2 which holds for all energies E so a solution of (1.340) is universal in E. For the electron it is known that there is no angular dependence of the particle charge density. For the proton there is only a weak angular dependence. Therefore we restrict the ∇^2 operator in (340) to the radial part, giving

$$\frac{d^2}{d\,r^2}\,\kappa^2\,(r)+\frac{2}{r}\,\frac{d}{d\,r}\,\kappa^2\,(r)=\kappa^4\,(r) \tag{1.366}$$

with

$$\kappa^2=\frac{2\,m\,(V-E)}{\hbar^2} \tag{1.367}$$

as before. When κ^2 is known, the potential is obtainable by

$$V=E+\frac{\hbar^2\,\kappa^2}{2\,m}. \tag{1.368}$$

In order to simplify Eq.(1.366) we substitute κ by a new function λ:

$$\lambda^2(r):=r\,\kappa^2(r). \tag{1.369}$$

This is the same procedure as getting rid of the first derivative in the standard solution procedure for the radial Schroedinger equation. Eq.(1.366) then reads:

$$\frac{d^2}{d\,r^2}\,\lambda^2\,(r)=\frac{\lambda^4\,(r)}{r}. \tag{1.370}$$

The initial conditions have to be chosen as follows. Because the radial coordinate in (1.369) starts at $r=0$, we have to use $\lambda^2(0)=0$ to be consistent. For the derivative of λ^2 follows from (1.369):

$$\frac{d\lambda^2}{dr}=\kappa^2+2\,r\,\frac{d\kappa}{dr}. \tag{1.371}$$

Only the first term contributes for $r = 0$ so that the initial value of κ^2 determines the derivative of λ^2 at this point. In total:

$$\lambda^2(0) = 0, \tag{1.372}$$

$$\frac{d\lambda^2}{dr}(0) = \kappa^2(0). \tag{1.373}$$

If $\kappa^2(0)$ is positive, we obtain only functions with positive curvature for λ^2 and κ^2, see Fig. 1.10. The potential function is always positive and greater than zero, allowing no bound states. Both functions diverge for large r. Therefore we have to start with a negative value of $\kappa^2(0)$. Then we obtain a negative region of the potential function, beginning with a horizontal tangent. This is the same as in the Woods Saxon potential, a model potential for of atomic nuclei. There is no singularity at the origin because there is no point charge.

Numerical studies give the result that the solutions λ^2 and κ^2 are always of the type shown in Fig. 1.11. The radial scale is determined by the depth of the inital value $\kappa^2(0)$. We have chosen this value so large that the radial scale (in atomic units) is in the range of the radii of elementary particles, see Table 1.1. As an artifact, the diverging behaviour for $r \to \infty$ found previously remains for negative initial values of the potential function. Obviously κ^2 crosses zero when the derivative of λ^2 has a horizontal tangent (Fig. 1.11). It would be convenient to cut the potential at this radius.

Solution of the radial Schroedinger equation

After having dertermined the potential function κ^2 which internally depends on E, we can solve the radial Schroedinger equation derived from (**??**):

$$-\frac{\hbar^2}{2\,m}\frac{d^2}{d\,r^2}\,\mathrm{R}\,(r) - \frac{\hbar^2}{m\,r}\frac{d}{d\,r}\,\mathrm{R}\,(r) + \mathrm{V}\,(r)\;\mathrm{R}\,(r) = E\,\mathrm{R}\,(r) \tag{1.374}$$

with R being the radial part of the wave function. We substitute R as usual:

$$P(r) := r\,R(r) \tag{1.375}$$

to obtain the simplified equation

$$\frac{d^2}{d\,r^2}\,\mathrm{P}\,(r) = \frac{2\,m}{\hbar^2}\;(\mathrm{V}\,(r) - E)\;\mathrm{P}\,(r)\,. \tag{1.376}$$

$V - E$ can be replaced by κ^2 which is already known from the constraint equation, so we have

$$\frac{d^2}{d\,r^2}\,\mathrm{P}\,(r) = \frac{\lambda^2\,\mathrm{P}\,(r)}{r} = \kappa^2\,\mathrm{P}\,(r)\,. \tag{1.377}$$

Obviously the energy parameter E is subsumed by κ. The computed κ function is valid for an arbitrary E. Since the left hand side of (1.377) is a replacement of the ∇^2 operator, the Schroedinger equation has been transformed into a Beltrami equation with variable scalar function κ^2 (assuming no divergence of P). There is no energy dependence left and the equation can be solved as an ordinary differential equation. This is a linear equation in P so that the result can be normalized arbitrarily and so can the final result R. This is the same again as for the solution procedure of the Schroedinger equation. Regarding the initial conditions, P starts at zero as discussed above and its derivative can be chosen arbitrarily, for example:

$$P(0) = 0, \tag{1.378}$$

$$\frac{dP}{dr}(0) = 1. \tag{1.379}$$

The results for R, R^2 and $R^2 r^2$ are graphed in Fig. 1.12. Again the functions have to be cut at the cut-off radius of about $2 \cdot 10^{-5}$ a.u.

Comparison with experiments

Experimental values of particle radii are listed in Table 1.1. The classical electron radius is calculated from equating the mass energy with the electrostostatic energy in a sphere and turns out to be simply

$$r_e = \alpha^2 a_0 \tag{1.380}$$

with α being the fine structure constant and a_0 the Bohr radius. This radius value is however larger than the proton radius. Therefore a more realistic calculational procedure seems to be scaling the proton radius with the mass ratio compared to the electron (second row in Table 1.1). The experimental limits are even smaller so that the accepted opinion is that the electron is a point particle which it certainly cannot be in a mathematical sense since there are no singularities in nature.

The charge density characteristics of proton and neutron are exponentially decreasing functions. This is not totally identical to the properties obtained for R^2 from our calculation (Fig. 1.13) which more looks like a Gaussian function. However, Gaussians have been observed for atomic nuclei containing more than one proton and neutron.

There is a diagram in the literature showing the charge densities for the proton and neutron [3] (replicated in Fig. 1.14). The charge densities start with zero values therefore they seem to describe the effective charge in a sphere of radius r which has to be compared with

$$\rho_e = R^2 \cdot r^2 \tag{1.381}$$

of our calculation. This function (with negative sign) has been graphed in Fig. 1.13 in the range below the cut-off radius. Since our function is not normalized the vertical scales differ. The proton has a shoulder in the charge density which is not reproduced by our calculation. The neutron is known not to be charge-neutral over the radius but to have a positive core and a negative outer region. The negative region which is called "shell" even pertains to the centre in Fig. 1.14. The shape of the shell is quite conforming to our calculation in Fig. 1.13. Some other experimental charge densities of the proton have been derived by Venkat et al. [4], see Fig. 1.4 therein. They compare quite well with our results for R^2r^2, Fig. 1.13 of this paper.

As already stated, our calculation does not contain an explicit energy parameter, therefore we do not obtain a mass spectrum of elementary particles or partons. The diameter of effective charge is defined by the initial value of κ^2. For the results shown we had to choose $\kappa^2 = -5 \cdot 10^{10}$ a.u. which is quite a lot. The rest energy of the proton is 938 MeV or $3.5 \cdot 10^7$ a.u. which is three orders of magnitude less. Obviously the potential has to be much deeper than the (negative) rest energy.

In conclusion, the Beltrami approach of ECE theory leads to a qualitatively correct desription of the internal structure of elementary particles, in particular the neutron. The binding energy cannot be determined since it cancels out from the calculation. It seems that the Beltrami structure is not valid in the boundary region of elementary particles or partons since the charge density does not go asymptotically to zero. This can be remedied by defining a cut-off radius where the radial function has a zero crossing. This was a first approach to compute the interior of elementary particles (the so-called parton structure) by ECE theory. For future developments more sophisticated approaches have to be found.

Particle	charge density characteristic	radius [m]	radius [a.u.]
electron (classical)	delta function	$2.82 \cdot 10^{-15}$	$5.33 \cdot 10^{-5}$
electron (derived)[a]	delta function	$9.1 \cdot 10^{-17}$	$1.72 \cdot 10^{-6}$
proton (measured)	neg. exponential function	$1.11 \cdot 10^{-15}$	$2.10 \cdot 10^{-5}$
proton (charge radius)	neg. exponential function	$8.8 \cdot 10^{-16}$	$1.66 \cdot 10^{-5}$
neutron (measuerd)	neg. exponential function	$1.7 \cdot 10^{-15}$	$3.21 \cdot 10^{-5}$
atomic nuclei	Gaussian or Fermi function	$2 - 8 \cdot 10^{-15}$	$4 - 15 \cdot 10^{-5}$

[a]Electron radius from volume comparison with $(m_{proton}/m_{electron})^{1/3}$

Table 1.1: Experimental data of elementary particles [3].

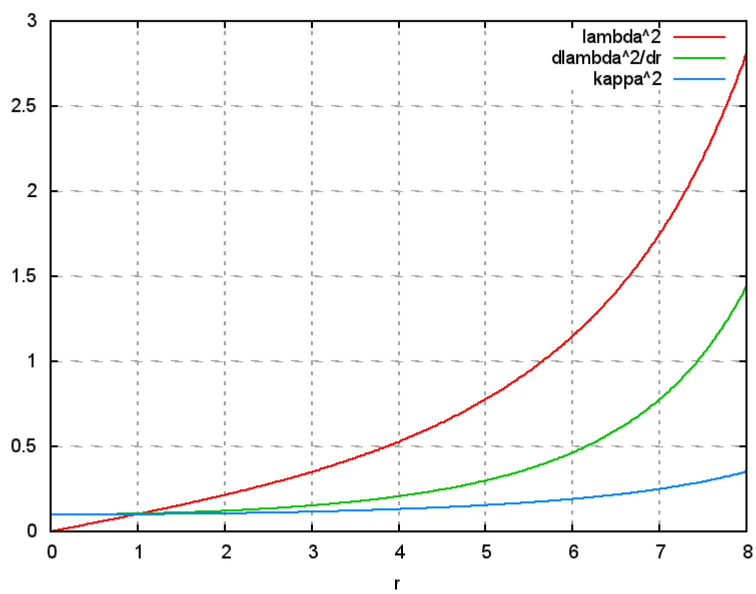

Figure 1.10: Solution functions of constraint equation (38) for $\kappa^2(0) > 0$.

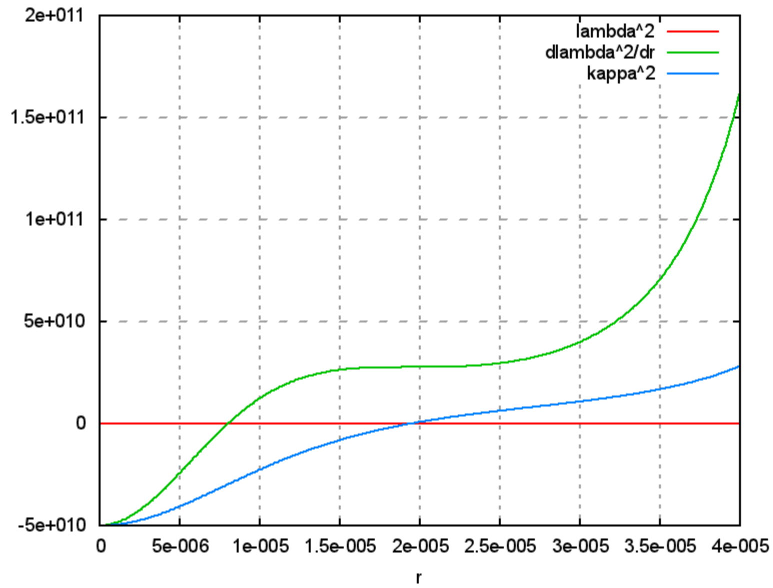

Figure 1.11: Solution functions of constraint equation (38) for $\kappa^2(0) < 0$.

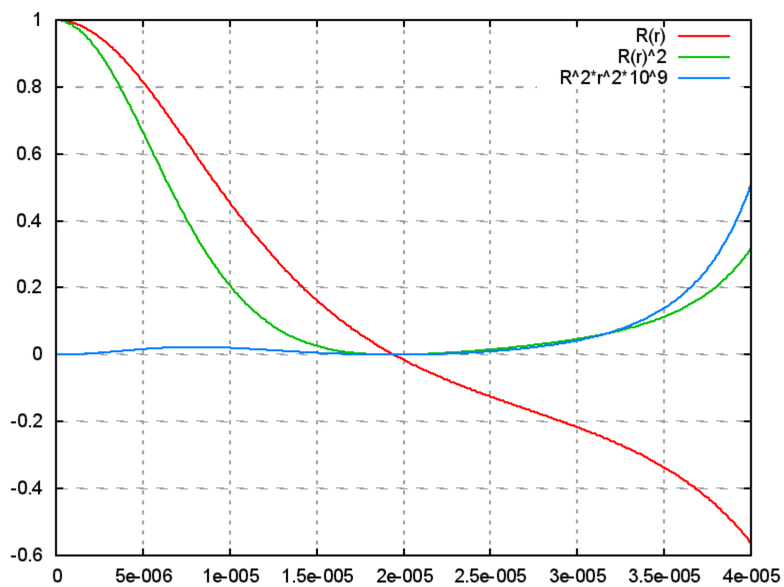

Figure 1.12: Parton solution of the Schroedinger equation.

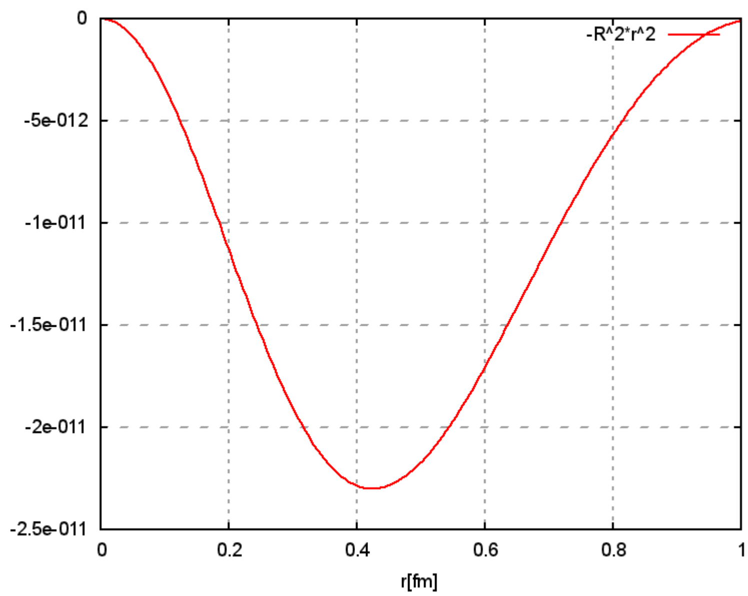

Figure 1.13: Radial wave function $-R^2 \cdot r^2$.

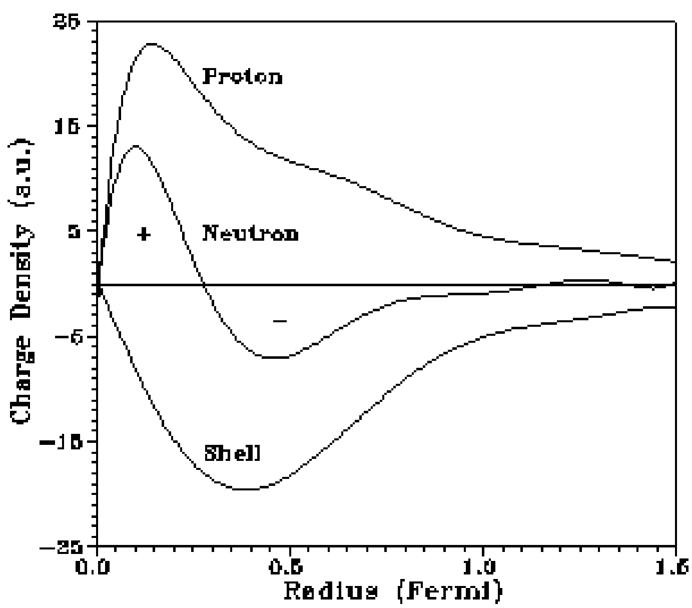

Figure 1.14: Experimental charge densities of elementary particles [3].

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[5] Proca