USE OF THE ROTATIONAL HOOKE LAW IN ORBITAL THEORY AND GENERAL RELATIVITY.

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ABSTRACT

It is shown that the orbital characteristics of a whirlpool galaxy are due to the rotational Hooke law, which gives a hyperbolic spiral. This inference should be regarded as a first attempt to go beyond line element general relativity, which is now known to be incorrect mathematically for all spherical spacetimes. The orbital characteristics of the solar system can be expressed as a non linear rotational Hooke law in a well defined approximation. This theory is related to the fully relativistic theory of orbits derived in the preceding paper of this series.

Keywords: Rotational Hooke law, ECE theory, theory of orbits.

UFT 197

1. INTRODUCTION

In recent papers of this series {1 - 10} it has been shown conclusively that the line element general relativity used by Einstein and followed dogmatically for nearly a century is trivially incorrect in all spherical spacetimes (UFT 194). This leaves the field equations of ECE theory as the only correct method of developing orbital theory within the philosophy of relativity. The field equations are developed in the engineering model on www.aias.us and other ECE sites. They are very useful in science and engineering but in this paper an entirely new method is explored of understanding all orbits in a simple way, using a simple law of physics. This effort should be regarded as a first step towards a new understanding of cosmology that is not beset by error as in Einsteinian general relativity, or by meaningless dogma such as dark matter, and this effort should be regarded as complementing the ECE field equations.

In the previous paper UFT 196 a relativistic theory of orbits was developed based on the spin connection and Cartan tetrad. That is a valid method of proceeding, a method based on the analytical function of the orbit deduced from astronomy. For example the orbit of a planet in the solar system was deduced to be an ellipse by Kepler, using the observations of Brahe. Later the ellipse was observed to be precessing, its perihelion moved by a tiny amount each year, so the orbit is a precessing ellipse whose analytical function is a simple one. Knowing this function the methods of UFT 196 are used in Section 2 of this paper to deduce the force law that describes the orbit in a theory of relativity based on the spin connection of Cartan, a theory that is free of error and dogma. The method can be extended to any orbit, and a few examples of spirals are given in this section, spirals that may describe whirlpool galaxies. Each has its own force law, so there is no "universal law of attraction" as in the Newtonian dogma.

In Section 3 the rotational Hooke law is introduced and developed in a classical context in order to show that the orbit of a whirlpool galaxy can be described by the law in a very simple way. This theory can be extended to a fully relativistic one based on the methods of UFT 196. It is also shown that the solar system orbits can be expressed as a non linear rotational Hooke law. This is a clear way of demonstrating that the orbit is driven by a torque, not by a linear force as in the Newtonian description. The torque is proportional to spacetime torsion.

Finally in Section 4, the characteristics of the various relativistic force laws of Section 2 are evaluated by computer.

2. FULLY RELATIVISTIC FORCE LAW FOR VARIOUS ORBITS.

In UFT 196 it was deduced that the force law for any orbit can be expressed in terms of a simple model for the spin connection ω and a characteristic interval of time t_g :

$$F(r) = -\frac{L^2}{mr^2}\left[\left(\frac{1}{1+\omega c t_g}\right)\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)+\frac{1}{r}\right) + \frac{1}{2} c t_g r^2 \left(\frac{\partial \omega}{\partial r}\right)\left(\frac{1}{1+\omega c t_g}\right)^2\left(\frac{1}{r^2}+\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2\right)\right] \quad -(1)$$

for a planar orbit in cylindrical polar coordinates (r, θ). The spin connection was modelled as:

$$\omega = -\frac{1}{r}, \quad \frac{\partial \omega}{\partial r} = \frac{1}{r^2} \quad -(2)$$

from experience in previous papers of this series {1 - 10}. More realistic models of the spin connection are known through the work of Eckardt and Lindstrom.{1}: Eq. (2) is used for the sake of illustration. From Eqs. (1) and (2):

$$F(r) = -\frac{L^2}{mr^2}\left[\left(1-\frac{ct_g}{r}\right)^{-1}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)+\frac{1}{r}\right) + \frac{1}{2}ct_g\left(1-\frac{ct_g}{r}\right)^{-2}\left(\frac{1}{r^2}+\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2\right)\right] \quad -(3)$$

and this equation is used in this section and numerically in Section 4 to analyse the force laws of various models of orbits.

The precessing ellipse for example is:

$$\frac{1}{r} = \frac{1}{\alpha}\left(1+\epsilon\cos(x\theta)\right) \quad -(4)$$

where 2α is the right latitude, ϵ the eccentricity, and x the precession constant. From Eq. (4):

$$\frac{d}{d\theta}\left(\frac{1}{r}\right) = -\frac{x\epsilon}{\alpha}\sin(x\theta), \quad -(5)$$

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = -\frac{x^2\epsilon}{\alpha}\cos(x\theta). \quad -(6)$$

Using these equations in Eq. (3) the force law has been evaluated numerically and is analysed in Section 4.

Various other orbits can be analysed similarly. For example the binary pulsar can be modelled as an inward spiralling and precessing ellipse:

$$\frac{1}{r} = \frac{1}{\alpha}e^{\beta\theta}\left(1+\epsilon\cos(x\theta)\right) \quad -(7)$$

and its force law worked out by computer. There are various kinds of whirlpool galaxy in which the stars are thrown out in a spiral pattern, for example in a hyperbolic spiral:

$$\frac{1}{r} = \frac{1}{r_0} \theta . \quad -(8)$$

The force equation must be constrained in this case by the observation of the velocity curve of the galaxy, an observation that shows that the velocity of the stars rises to a constant with increasing r and remains constant. This behaviour is non Einsteinian completely, and also completely non Newtonian. It has been explained however in this series of ECE papers. If the velocity is constant there is no force, so this observational constraint means that the force must go to zero for large r. This is in turn a constraint on the spin connection and characteristic time interval t_g . These matters are discussed further in Section 4.

The analysis may be repeated for various kinds of spirals, for example the logarithmic spiral:

$$\frac{1}{r} = \frac{1}{r_0} \exp(-\beta\theta), \quad -(9)$$

the Archimedes spiral:

$$r = r_1 + \theta r_0 , \quad -(10)$$

Fermat's spiral:

$$\frac{r_0}{r} = \frac{1}{\theta^{1/2}} , \quad -(11)$$

the lituus:

$$\frac{r_0}{r} = \theta^{1/2} , \quad -(12)$$

and Euler's double spiral:

$$B(r) = S(r) + iC(r) \quad -(13)$$

where:

$$C(r) = \int_0^r \cos\left(\frac{1}{2}\pi x^2\right) dx \quad -(14)$$

and

$$S(r) = \int_0^r \sin\left(\frac{1}{2}\pi x^2\right) dx \quad -(15)$$

are Fresnel's integrals. A double spiral galaxy may or may not have been observed, but any type of galaxy can be modelled functionally and its force law found.

In this context the rotational Hooke law is:

$$T_q = -k\theta \quad -(16)$$

where k is a constant and T_q is the torque (not to be confused with the torsion). It is seen immediately from Eq. (16) that the reason for a spiral pattern of stars is torque in the rotational Hooke law, a spiral law. It is an Archimedes spiral with:

$$r_1 = 0, \quad r_1 \rightarrow k. \quad -(17)$$

In the spiral itself r is proportional to $-\theta$, in the rotational Hooke law the torque is proportional to $-\theta$.

3. CLASSICAL ORBITAL THEORY AND THE ROTATIONAL HOOKE LAW.

In the classical theory of orbits, the lagrangian is:

$$\mathcal{L} = \frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\theta}^2 \right) - U(r) \quad -(18)$$

and the constant total angular momentum is:

$$L = m r^2 \frac{d\theta}{dt} . \quad -(19)$$

As in UFT 119 on www.aias.us the magnitude of torque may be defined as:

$$Tq = \omega L \quad -(20)$$

where the angular velocity is:

$$\omega = \frac{d\theta}{dt} . \quad -(21)$$

Therefore:

$$Tq = L \frac{d\theta}{dt} = \frac{L^2}{m r^2} . \quad -(22)$$

The orbit given by the rotational Hooke law is therefore:

$$Tq = -k\theta = \frac{L^2}{m r^2} , \quad -(23)$$

which is a hyperbolic spiral of the type observed in a whirlpool galaxy of stars. This classical analysis shows immediately that torsion is responsible for the pattern of stars in a whirlpool galaxy, not Einsteinian relativity, dark matter or Newtonian dynamics.

For the sake of illustration the analysis may be extended to the Newtonian orbit, which is an ellipse:

$$\frac{1}{r^2} = \frac{1}{d^2}\left(1 + \epsilon \cos\theta\right)^2. \quad -(24)$$

So the torque from Eq. (24) is:

$$Tq = \frac{L^2}{md^2}\left(1 + \epsilon \cos\theta\right)^2 \quad -(25)$$

and is not a simple Hooke law. The latter however is derived in a linear approximation similar to the law of springs, the original Hooke law. The law of springs is non linear in general. For small angles:

$$Tq \sim \frac{L^2}{md^2}\left(1 + \epsilon\left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots\right)\right)^2 \quad -(26)$$

and for a nearly circular orbit:

$$\epsilon \sim 1 \quad -(27)$$

so the torque is approximated by:

$$Tq \sim \frac{4L^2}{md^2} - \frac{L^2\theta^2}{md^2} \quad -(28)$$

which is a simple non linear expression in angular displacement θ. This simple classical analysis can be extended relativistically using the methods of UFT 196, and is used to show that the orbit may be thought of entirely in terms of torque.

4. NUMERICAL ANALYSIS OF VARIOUS ORBITAL FORCE LAWS

Section by Dr. Horst Eckardt

Use of the rotational Hooke Law in orbital theory and General Relativity

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4 Numerical analysis of various orbital force laws

The force laws for the orbitals described in section 2 have been evaluated by computer. The general procedure was as follows:

1. define the orbit $r(\theta)$,
2. compute the terms $\frac{d}{d\theta}\frac{1}{r}$ and its second derivative,
3. substitute these terms in Eq.(3),
4. replace the occurences of the θ variable in E.(3) by r by using the definition of $r(\theta)$.

This worked in nearly all cases. The results are valid for the spin connection of the form

$$\omega = -\frac{1}{r} \tag{29}$$

(see Eq.(2)) and similar for all orbits except for the binary pulsar.

Solar system

First we investigate the precessing ellipse given by

$$r = \frac{\alpha}{1 + \epsilon \cos(x\,\theta)}. \tag{30}$$

The resulting force is

$$F = -\frac{\left(\left(\left(\left(c\,\epsilon^2 - c\right)\,t_f + 2\,\alpha\right)\,r^2 - 2\,\alpha^2\,r + \alpha^2\,c\,t_f\right)\,x^2 + 2\,\alpha^2\,r - \alpha^2\,c\,t_f\right)\,L^2}{2\,\alpha^2\,m\,r^2\,\left(r - c\,t_f\right)^2} \tag{31}$$

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The curve $F(r)$ is shown for different values of the cosmological parameter t_f in Fig. 1. It can be seen that the occurence of this parameter shifts the force center from $r = 0$ to finite values of r. In Fig. 2 this curve is plotted for $t_f = 0.6$ in a small radius range. There is a pole of $F(r)$. The parameters are chosen as specified in the figure captions. The question is which values of t_f are realistic in case of the solar system. We chose the parameters for the earth orbit as

$$\begin{aligned} m &= 5.9742 \cdot 10^{24} kg, \\ L &= 2.663 \cdot 10^{40} kg\ m^2, \\ \alpha &= 1.4960 \cdot 10^{11} m, \\ x &= 1, \\ \epsilon &= 0.0167. \end{aligned} \tag{32}$$

Fig. 3 shows the resulting absolute values of the gravitational force for various values of t_f. It has to be noted that $c\ t_f$ is the way that light travels in t_f seconds. Since the Newtonian force law works relatively precisely in the solar system, we can assume that the singularity produced at the sphere with radius $c\ t_f$ is positioned in the interior of the sun which has a radius of $6.955 \cdot 10^8 m$. Therefore we can estimate

$$c\ t_f < 6.955 \cdot 10^8 m \tag{33}$$

or

$$t_f < 2.3s. \tag{34}$$

This is a quite small value compared to cosmological time scales. The value might be larger for heavy stars, for example supermassive objects in the center of galaxies. Therefore a time evolution of the universe could be possible which increases the gravitational forces, if t_f is interpreted in this way. This would be the prediction of time-varying laws of nature. Since the term $c\ t_f$ comes from the kinetic energy, it would mean that this energy changes over time, for example by taking up energy from the background or vacuum field. The singularity, if coming to lie outside of stars, would play the role of a horizon formerly attributed to black holes which have been shown not to exist.

Binary pulsar

For binary pulsars the orbit is a decreasing pressessing ellipse of the form

$$r = \frac{\alpha \exp(-\beta\ \theta)}{1 + \epsilon \cos(x\ \theta)} \tag{35}$$

where β is a decay constant. In this case it is not possible to substitute θ completely in the force law. The resulting formula is quite complex:

$$\begin{aligned} F = &-\frac{L^2}{2\,\alpha^2\, m\, r^2\, (r - c\, t_f)^2} \\ &\cdot \Big(\left(\left(c\,\epsilon^2 - c\right)\, t_f\, r^2\, e^{2\,\beta\,\theta} + 2\,\alpha\, r^2\, e^{\beta\,\theta} - 2\,\alpha^2\, r + \alpha^2\, c\, t_f \right)\, x^2 \\ &+ (2\,\alpha\,\beta\, c\, t_f - 4\,\alpha\,\beta\, r)\, \sqrt{\left(\epsilon^2\, r^2 - r^2\right)\, e^{2\,\beta\,\theta} + 2\,\alpha\, r\, e^{\beta\,\theta} - \alpha^2}\, x \\ &+ \left(2\,\alpha^2\,\beta^2 + 2\,\alpha^2\right)\, r + \left(-\alpha^2\,\beta^2 - \alpha^2\right)\, c\, t_f \Big). \end{aligned} \tag{36}$$

In the resulting plot (Fig. 4) it can be seen that there is a common maximum radius for the force for all t_f's. Above this radius the expression for the force becomes complex because the square root term becomes imaginary. This may indicate that there is a maximum radius for a binary pulsar and the orbit is always shrinking.

Hyperbolic spiral

The hyperbolic spiral is defined by

$$r = \frac{a}{\theta} \tag{37}$$

with a characteristic radius a. Application of the above algorithm yields

$$F = -\frac{\left(c\,t_f\,r^2 + 2\,a^2\,r - a^2\,c\,t_f\right)\,L^2}{2\,a^2\,m\,r^2\,\left(r - c\,t_f\right)^2}. \tag{38}$$

This is a force law of orders $1/r^2$ to $1/r^4$. There is no angular dependence. The graph looks very similar to Figs. 1 and 2.

Logarithmic spiral

The logarithmic spiral is defined by an exponential angular dependence

$$r = r_0 \exp(\beta\theta) \tag{39}$$

with a characteristic radius r_0. Similarly as for the hyperbolic spiral, the force law is

$$F = -\frac{\left(\beta^2 + 1\right)\,\left(2\,r - c\,t_f\right)\,L^2}{2\,m\,r^2\,\left(r - c\,t_f\right)^2}. \tag{40}$$

This is a force law of orders $1/r^3$ and $1/r^4$. Again the graph looks very similar to Figs. 1 and 2.

Archimedes spiral

Another type of spiral is the Archimedes spiral

$$r = a + b\theta \tag{41}$$

with characteristic lenght parameters a and b. The calculational method leads to the force law

$$F = -\frac{\left(2\,r^3 - c\,t_f\,r^2 + 4\,b^2\,r - 3\,b^2\,c\,t_f\right)\,L^2}{2\,m\,r^4\,\left(r - c\,t_f\right)^2} \tag{42}$$

which is even up to orders of $1/r^6$. Nevertheless the graph looks similar to Figs. 1 and 2.

Fermat's spiral

Similar results hold for Fermat's spiral

$$r = a\sqrt{\theta}. \tag{43}$$

The force law

$$F = -\frac{\left(8\,r^5 - 4\,c\,t_f\,r^4 + 6\,a^4\,r - 5\,a^4\,c\,t_f\right)\,L^2}{8\,m\,r^6\,\left(r - c\,t_f\right)^2} \tag{44}$$

is of maximum order $1/r^8$.

The Lituus

No much different results are obtained for the Lituus

$$r = a\frac{1}{\sqrt{\theta}}. \tag{45}$$

The force law is

$$F = \frac{\left(2\,r^5 - 3\,c\,t_f\,r^4 - 8\,a^4\,r + 4\,a^4\,c\,t_f\right)\,L^2}{8\,a^4\,m\,r^2\,\left(r - c\,t_f\right)^2} \tag{46}$$

and contains a term proportional to r which lets the force raise above zero for large r values but otherwhere is similar to Figs. 1 and 2 again. The maximum order of the denominator terms is $1/r^6$.

Euler's spiral

Finally we investigate Euler' spiral which is interesting because it consists of two spirals which are connected and the common arm has zero curvature at the position of point symmetry. Only one of the spirals is shown in Fig. 5. Its curvature changes linearly with curve length. Euler spirals are also commonly referred to as spiros, clothoids or Cornu spirals.

In cartesian coordinates the spiral is given in normlized, parametric form by

$$S_x = x(t) = \int_0^t \cos\left(s^2\right)\,ds, \tag{47}$$

$$S_y = y(t) = \int_0^t \sin\left(s^2\right)\,ds \tag{48}$$

where t is a parameter and s an integration variable. The integrals are known as Fresnel integrals and cannot be solved analytically. There are a series expansion and some approximative formulas. We use the former:

$$S_x = \sum_{n=0}^{\infty}(-1)^n\frac{t^{4n+1}}{(2n)!\,(4n+1)}, \tag{49}$$

$$S_y = \sum_{n=0}^{\infty}(-1)^n\frac{t^{4n+3}}{(2n+1)!\,(4n+3)}. \tag{50}$$

Since we have to transform the spiral to polar coordinates we shift the coordinate origin to the center which is a shift of

$$S_x \to S_x - \frac{\pi}{8}, \tag{51}$$

$$S_y \to S_y - \frac{\pi}{8}. \tag{52}$$

For small values of t the series converges rapidly but for larger t values, which corresponds to the inner circular parts, convergence is slow. We needed $n = 80$ to acheive convergence in the range through $t = 5.6$ as shown in Fig. 5.

Since the spiral is given numerically we have to evaluate the force law numerically too. First we transform to polar coorcinates:

$$r = \sqrt{S_x^2 + S_y^2}, \tag{53}$$

$$\theta = \arctan \frac{S_y}{S_x}. \tag{54}$$

Then we compute the derivatives which are needed for the force law from the chain rules

$$\frac{dr}{d\theta} = \frac{dr}{dt}\frac{dt}{d\theta} = \frac{dr}{dt}\left(\frac{d\theta}{dt}\right)^{-1}, \tag{55}$$

$$\frac{d^2r}{d\theta^2} = \frac{d^2r}{dt^2}\left(\frac{dt}{d\theta}\right)^2 + \frac{dr}{dt}\frac{d^2t}{d\theta^2}. \tag{56}$$

The derivatives are computed numerically by the usual discrete difference schemes and inserted in the original force equation (3):

$$F = \frac{\left(2\,r\,(r - c\,t_f)\,\left(\frac{d^2}{d\,\theta^2}\,r\right) - (4\,r - 3\,c\,t_f)\,\left(\frac{d}{d\,\theta}\,r\right)^2 - r^2\,(2\,r - c\,t_f)\right)\,L^2}{2\,m\,r^4\,(r - c\,t_f)^2}. \tag{57}$$

The force law is graphed in Fig. 6 in polar coordinates, again for four values of t_f. It can be seen that the forces are regular spirals, there is no singularity in this case. Increasing t_f leads to an increase of the force at the the same angular values. This means, the orbit has a smaller radius for growing t_f since the atrractive force is greater. This is similar in behaviour to the other spiral types. In total only the binary pulsar behaves different from the ellipse and various types of spirals.

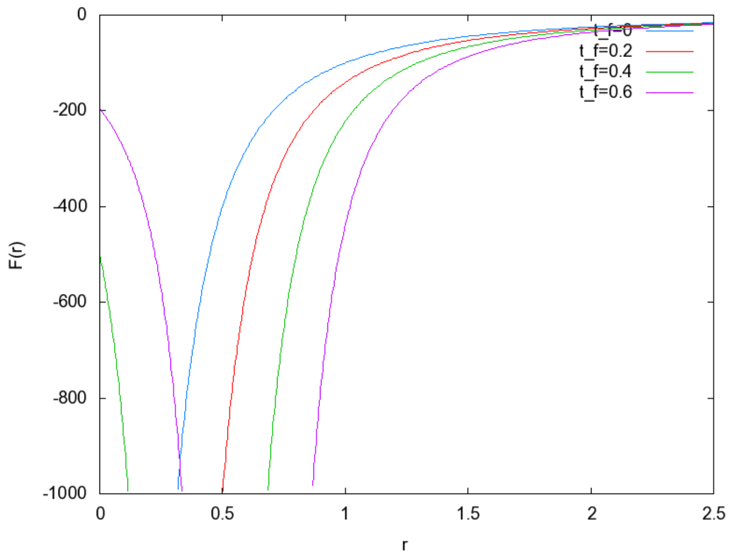

Figure 1: Force law for the solar system for different parameters t_f with $m = c = \alpha = x = 1,\ L = 10,\ \epsilon = 0.1$.

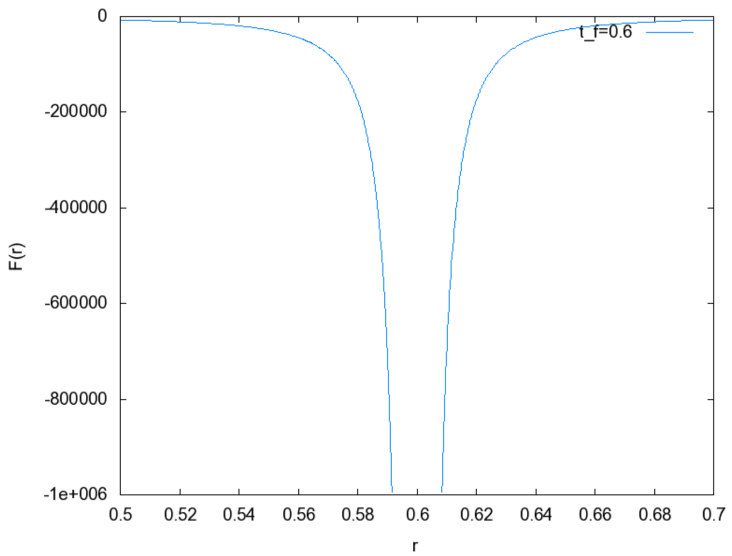

Figure 2: Force law for the solar system, radius section near to singularity.

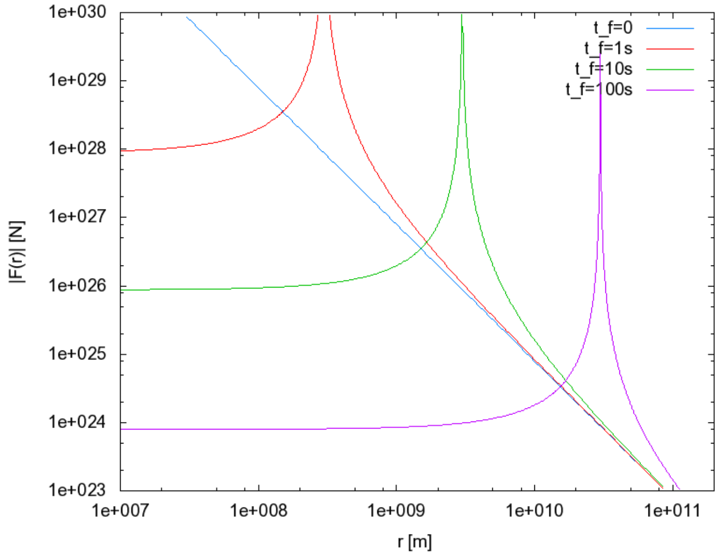

Figure 3: Force law for the sun-earth system for different parameters t_f, logarithmic scales.

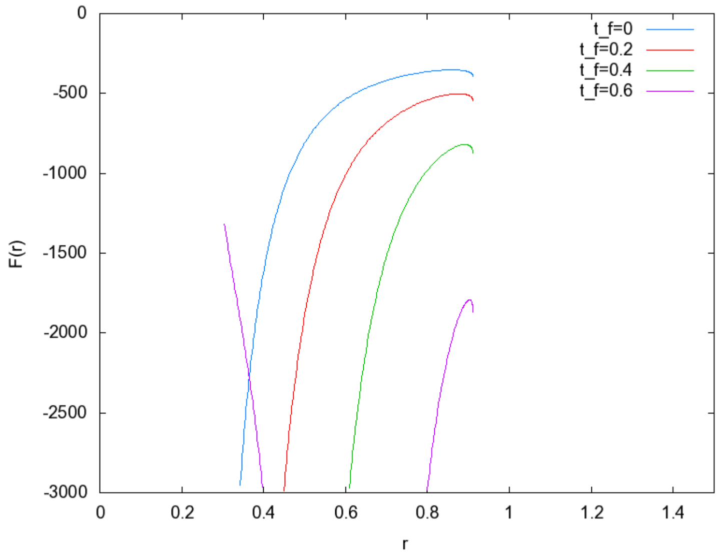

Figure 4: Force law for the binary pulsar for different parameters t_f with $m = c = \alpha = \beta = x = 1,\ L = 10,\ \theta = \pi/4$.

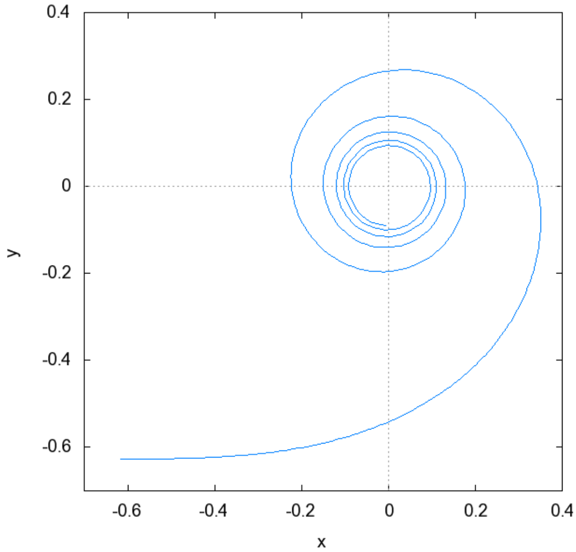

Figure 5: Normalized Euler spiral with shifted coordinates.

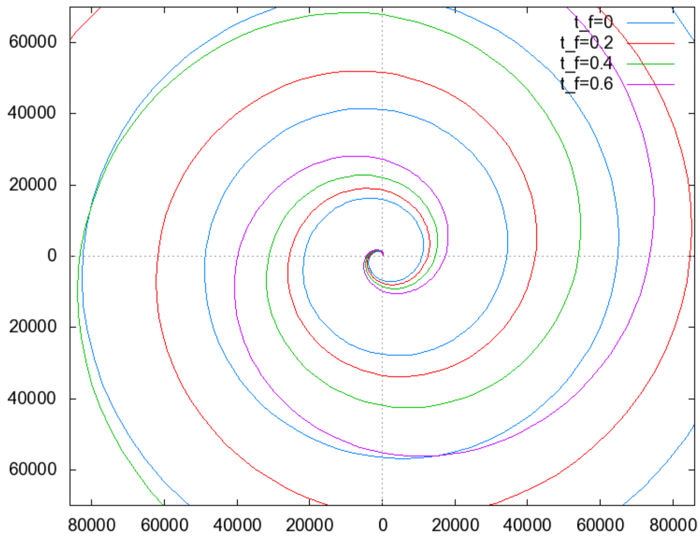

Figure 6: Force law of Euler spiral (absolute values).

ACKNOWLEDGMENTS

The British Government is thanked for the award of a Civil List Pension (2005) and Armiger rank in 2008. The staff of AIAS and many others are thanked for many interesting discussions and voluntary work on behalf of science: David Burleigh for posting, Robert Cheshire, Alex Hill and Simon Clifford for broadcasting, and Alex Hill for translation and broadcasting. The AIAS is established under the aegis of the Newlands Family Trust established in 2011.

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