# EFFECT OF PHOTON MASS ON THE RAYLEIGH JEANS DENSITY OF STATES AND THE PLANCK DISTRIBUTION.

by

M. W. Evans, H. Eckardt, G. J. Evans and T. Morris,

Civil List, AIAS and UPITEC

(www.webarchive.org.uk , www.aias.us, www.atomicprecision.com, www.upitec.org, www.et3m.net)

ABSTRACT

The Rayleigh Jeans density of states is corrected for the rest mass of the photon, and the Planck distribution and Stefan Boltzmann law calculated for photons with mass. The number of massive photons in the observable universe is calculated using the missing mass and volume of the observable universe. It is deduced that missing mass can be accounted for entirely by the mass of photons propagating with velocities ranging from close to the speed of light to zero. Therefore it is unnecessary to postulate the existence of dark matter.

Keywords: ECE theory, massive photon, Rayleigh Jeans density of states and Planck distribution.

UFT 310

## 1. INTRODUCTION

In recent papers of this series {1 - 12} new and rigorous tests of the quantum theory have been deduced by a simple comparison of the Planck distribution with the Beer Lambert law. The quantum theory shows that the frequency of a probe laser is shifted in a manner governed by the transition dipole moment of the power absorption coefficient. This work developed following upon the experimental discovery by G. J. Evans and T. Morris of repeatable and reproducible shifts in electromagnetic frequencies in visible frequency light propagating through liquids and other materials. These are the Evans / Morris shifts, and are fundamental to optics and spectroscopy. In UFT308 these shifts were evaluated for rotational spectra in the microwave and far infra red, and in UFT309 the theory was developed for radiation and particle scattering.

In Section 2 the fundamental Rayleigh Jeans density of states is developed with a finite photon rest mass by using the Proca equation {1 - 12} instead of the d'Alembert equation as used by Rayleigh. The calculations are carried out for monochromatic and polychromatic radiation, in which case the Stefan Boltzmann law is corrected for photon rest mass. The latter is estimated from the de Broglie equation and from the lowest observable electromagnetic frequencies. These are sub hertzian, so the photon rest mass is less than about $10^{-51}$ kgm. In consequence its effect on the Stefan Boltzmann law is negligible. The calculation is repeated to find the number of photons N in the observable universe. Using the missing mass M of the observable universe a photon mass m can be estimated by assuming that M is mN. The result is $5.3 \times 10^{-40}$ kg, and is orders of magnitude heavier than the rest mass $m_0$ of the photon. Therefore m is interpreted as $\gamma m_0$, where $\gamma$ is the Lorentz factor. The velocity in the latter is the mean velocity of massive photons propagating with velocities that range from close to c to zero.

This paper should be read with its background notes posted with UFT310 on www.aias.us. Note 310(1) compares the Planck oscillator theory with Fermi Dirac and Bose Einstein statistics in order to give the background theory for the later development of the Planck distribution for an electron gas. This can be used to give the Evans / Morris effects of en electron beam interacting with a sample using the generalized Beer Lambert law developed in UFT309. Note 310(2) gives the background to the original 1900 density of states calculation by Rayleigh, using the d'Alembert wave equation and massless photons. It is well known that Rayleigh assumed two states of polarization and first calculated the density of states as the number of massless photons in a given volume of monochromatic radiation. He proceeded to calculate the density of states for polychromatic radiation through the expression dN / V, where dN is an infinitesimal. In so doing he neglected higher order infinitesimals. As shown in UFT291 this procedure is incorrect, and the correct result was given in that paper. Note 310(2) proceeds to correct the Rayleigh calculation in another way, using photon mass, and this calculation begins Section 2. Note 310(3) gives details of the calculation of the Planck distribution for bosons and fermions with mass, and these calculations are summarized in Section 2. In ECE theory the graviton is also a boson with mass. Note 310(4) gives details of the calculation of Evans / Morris effects for an electron beam interacting with matter, using the Planck distribution for electrons calculated in Note 310(3). Note 310(5) gives details of the calculation of the Stefan Boltzmann law for massive photons, and derives a method for the direct experimental measurement of the photon rest mass from measurement of the flux density in watts per square metres of a monochromatic beam such as a laser and its angular frequency. This result is used in Section 2 and the calculations of this note are developed in Section 3 using numerical analysis and graphics. Note 310(6) gives complete details of the correction of the Stefan Boltzmann law due to photon mass and the result is used in Section 2. Note 310(7) gives complete details of the calculation of the number of photons with rest mass $m_0$ in the observable universe. This calculation proceeds by considering the background radiation as a black body radiator at about 2.7 K, and uses the volume of the observable universe. Using the missing mass of the universe gives an estimate of the average mass of moving photons in the universe. The result is

$$m = \gamma m_0 = 5.3 \times 10^{-40} \text{ kg} \quad -(1)$$

orders of magnitude heavier than the rest mass of about

$$m_0 = 5.1 \times 10^{-51} \text{ kg} \quad -(2)$$

estimated from the lowest observable electromagnetic frequencies and de Broglie equation. This note reviews other estimates of photon mass from UFT150, 155, 244, 245, 264, 279 and 305. All these estimates vary with the photon velocity v and are all much higher than the photon rest mass. Therefore m is the mass of a moving photon and $m_0$ is the mass of a photon at rest. The missing mass of the universe is made up entirely of photons propagating from close to c to zero. Finally Note 310 (8) gives complete details of the self consistent calculation of the mass m, interpreted as the average mass of all the photon in the observable universe. This is multiplied by the number of photons in the universe to give the missing mass of the universe. This note gives a Beer Lambert law for photons with mass among other deductions. There is no need to postulate "dark matter".

## 2. RAYLEIGH JEANS DENSITY OF STATES AND PLANCK DISTRIBUTION FOR A PHOTON WITH MASS.

In his original 1900 calculation Rayleigh used the d'Alembert wave equation for a massless photon as detailed in the introduction and background notes. This wave equation can be written as:

$$\square f = 0 \quad -(3)$$

where f is a wave function. It is derived from the ECE wave equation in Note 310(2). In the presence of photon rest mass $m_0$, Eq. ( 3 ) is replaced by the 1938 Proca equation:

$$\left(\square + \left(\frac{m_0 c}{\hbar}\right)^2\right) f = 0 \quad -(4)$$

which is a limit of the ECE wave equation. Here c is the speed of light in vacuo and $\hbar$ the reduced Planck constant. Note carefully that in the Proca equation c is not the photon velocity, it is defined as a constant of special relativity, the upper bound on the velocity of the photon. This interpretation was proposed by Poincaré in 1905. The photon is treated in the same way as any relativistic particle.

Eq. ( 4 ) is the quantized equivalent of the Einstein energy equation:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(5)$$

with:

$$E = \hbar\omega, \quad \underline{p} = \hbar \underline{k}. \quad -(6)$$

So the relation between wavenumber and frequency becomes:

$$c^2 k^2 = \omega^2 - \omega_0^2 \quad -(7)$$

where $\omega_0$ is the rest frequency of the photon:

$$\omega_0 = m_0 c^2 / \hbar. \quad -(8)$$

Rayleigh used:

$$m_0 = 0 \quad -(9)$$

so he used:

$$ck = \omega. \quad -(10)$$

Therefore to correct the Rayleigh Jeans density of states for photon rest mass, $m_0$, the quantity $\omega^2$ is replaced everywhere by $\omega^2 - \omega_0^2$.

As shown in detail in Note 310(3), the number density for monochromatic radiation is

$$\frac{N}{V} = \frac{1}{3c^3\pi^2}\left(\omega^2 - \omega_0^2\right)^{3/2}. \quad -(11)$$

The Planck distribution in this case is:

$$\frac{E}{V} = \frac{\hbar\omega}{3c^3\pi^2}\frac{\left(\omega^2 - \omega_0^2\right)^{3/2}}{e^y - 1} \quad -(12)$$

where the mean energy of an oscillator is:

$$\langle \hbar\omega \rangle = \frac{\hbar\omega}{e^y - 1} \quad -(13)$$

where:

$$\hbar\omega = \gamma mc^2. \quad -(14)$$

The Lorentz factor is:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(15)$$

where v is the photon velocity. In Eq. ( 13 ):

$$y = \frac{\hbar\omega}{kT} \quad -(16)$$

where k is the Boltzmann constant and where T is the temperature.

Therefore the Planck distribution for an ensemble of massive photons at the monochromatic frequency $\omega$ is:

$$\frac{E}{V} = \frac{\hbar\omega}{3c^3\pi^2} \frac{\left(\omega^2 - \omega_0^2\right)^{3/2}}{e^y - 1} \quad -(17)$$

in joules per cubic metre. The flux density is:

$$\Phi = c \frac{E}{V} \quad -(18)$$

in watts per square metre.

For polychromatic radiation:

$$\frac{dN}{V} = \frac{1}{3c^3\pi^2} \left( \left( (\omega + d\omega)^2 - \omega_0^2 \right)^{3/2} - \left( \omega^2 - \omega_0^2 \right)^{3/2} \right) \quad -(19)$$

so:

$$\frac{dE}{V} = \langle \hbar\omega \rangle \frac{dN}{V} \quad -(20)$$

and the total flux density of a polychromatic ensemble of massive photons is:

$$\frac{E}{V} = \frac{1}{V} \int dE. \quad -(21)$$

As shown in UFT291, the higher order infinitesimals cannot be neglected.

The Planck distribution for a monochromatic ensemble of fermions with mass is:

$$\frac{E}{V} = \frac{\hbar\omega}{3c^3\pi^2} \frac{\left(\omega^2 - \omega_0^2\right)^{3/2}}{e^y + 1} \quad -(22)$$

using Fermi Dirac statistics detailed in Notes 310(1) and 310(2). For both the massive boson and massive fermion:

$$E = \hbar\omega = \gamma m c^2 \quad -(23)$$

$$\underline{p} = \hbar\underline{\kappa} = \gamma m \underline{v} \quad -(24)$$

and the calculation can be developed in terms of the photon or fermion velocity as in Note 310(3). From these calculations it is possible to show as in Note 310(4) that the Evans Morris effect for a monochromatic electron beam interacting with matter is:

$$\frac{\Phi}{\Phi_0} = \exp(-\alpha Z) = \frac{\omega}{\omega_0}\left(\frac{\omega^2-\omega_r^2}{\omega_0^2-\omega_r^2}\right)^{3/2}\left(\frac{e^{y_0}+1}{e^{y}+1}\right) \quad -(25)$$

where the generalized Beer Lambert law of UFT309 is used for an electron beam:

Summarizing Note 310(5), the Stefan Boltzmann law may be corrected for photon mass by accepting Rayleigh's original 1900 density of states:

$$\frac{dN}{V} = \frac{\omega^2}{c^3\pi^2}\, d\omega \quad -(26)$$

as a starting point. This is incorrect as described in UFT291 because of Rayleigh's neglect of higher order infinitesimals, but is used to illustrate the correction for photon mass. The rigorous correction must be based on Eq. ( 19 ). Eq. ( 26 ) is corrected by photon mass to:

$$\frac{dN}{V} = \frac{\Omega^2}{c^3\pi^2}\, d\Omega \quad -(27)$$

where:

$$\underline{\Omega} = (\omega^2 - \omega_0^2)^{1/2} \quad -(28)$$

Therefore:

$$\frac{d\Omega}{d\omega} = \frac{\omega}{(\omega^2 - \omega_0^2)^{1/2}} \quad -(29)$$

and the Rayleigh density of states corrected for photon rest mass is:

$$\frac{dN}{V} = \frac{\omega}{c^3 \pi^2} \left(\omega^2 - \omega_0^2\right)^{1/2} d\omega. \quad -(30)$$

It follows as in Note 310(5) that the Stefan Boltzmann law corrected for photon rest mass is:

$$\frac{E}{V} = \frac{\hbar}{c^3 \pi^2} \int_0^\infty \frac{\omega^2 (\omega^2 - \omega_0^2)^{1/2}}{e^y - 1} d\omega. \quad -(31)$$

This correction is discussed in Section 3 using numerical and graphical analysis. The integrals in Eq. ( 31 ) can be worked out analytically as in Note 310(6) to give:

$$\frac{E}{V} = \left(\frac{\pi^2 k^4}{15 c^3 \hbar^3}\right) T^4 - \left(\frac{k^2 c}{12 \hbar^3}\right) m^2 T^2. \quad -(32)$$

Therefore the photon rest mass can be worked out from the measured flux density ( $\Phi$ ) of black body radiation at a temperature T:

$$m^2 = \left(\frac{12 \pi^2 k^2}{15 c^4}\right) T^2 - \left(\frac{12 \hbar^3}{k^2 c^2 T^2}\right) \Phi \quad -(33)$$

This is an experiment that has been carried out in standards laboratories. For a monochromatic beam the photon rest mass can be worked out from the flux density of the monochromatic Planck distribution corrected for photon mass:

$$\Phi = \frac{cE}{V} = \frac{\hbar (\omega^2 - \omega_0^2)^2}{3 c^2 \pi^2 (e^y - 1)}. \quad -(34)$$

The photon rest mass is given by:

$$\omega_0^2 = \left(\frac{m_o c^2}{\hbar}\right)^2 = \omega^2 - \left(\frac{3c^2\pi^2}{\hbar}\left(\exp\left(\frac{\hbar\omega}{kT}\right)-1\right)\bar{I}\right)^{1/2} \quad -(35)$$

so all that is required to find it is the flux density of a laser and its frequency. These must be measured with a sufficiently high accuracy because $m_0$ is less than about $10^{-51}$ kg..

Any deviation from the Stefan Boltzmann law is due to the photon rest mass $m_0$.

The number of photons in the universe can be calculated by assuming that the background radiation at 2.7 K is a black body radiator. Its density of states corrected for photon mass is:

$$\frac{dE}{V} = \frac{\hbar}{c^3\pi^2}\,\frac{\omega^2\left(\omega^2-\omega_0^2\right)^{1/2}}{e^y-1} \quad -(36)$$

This energy is generated by N photons each of energy $\hbar\omega$, so

$$dN = \frac{dE}{\hbar\omega} \quad -(37)$$

and:

$$\frac{N}{V} = \int_0^\infty \frac{\omega^2-\omega_0^2}{\pi^2c^3\left(e^y-1\right)}\,d\omega. \quad -(38)$$

The integrals are worked out with:

$$\int_0^\infty \left(\frac{x^{2n}}{e^x-1}\right)dx = (2n)!\,\zeta(2n+1) \quad -(39)$$

where $\zeta$ is the Riemann zeta function. To an excellent approximation:

$$\frac{N}{V} = \frac{1}{\pi^2c^3}\int_0^\infty \frac{\omega^2}{e^y-1}\,d\omega = \frac{2\zeta(3)}{\pi^2}\left(\frac{k}{c\hbar}\right)^3 T^3 \quad -(40)$$

where the third zeta function is:

$$\zeta(3) = 1.2026. \quad -(41)$$

The received opinion is that the volume of the observable universe is:

$$V = 4 \times 10^{83} \, m^3 \quad -(42)$$

so the number of photons radiated in this volume by a background temperature of 2.7 K is:

$$N = 1.6 \times 10^{92}. \quad -(43)$$

The received opinion is that the mass of the observable universe is:

$$M = 10^{53} \, kg \quad -(44)$$

of which 84.5% is missing mass due to discrepancies between between the mass of objects determined from gravitational effects and the mass determined form the matter they contain, such as stars, gas and dust. So the missing mass is:

$$M_1 = 8.5 \times 10^{52} \, kg. \quad -(45)$$

Assume that:

$$M_1 = mN \quad -(46)$$

where m is the mean mass of a moving photon in the balck body radiation that pervades the universe due to a background temperature of 2.7 K, then:

$$m = 5.3 \times 10^{-40} \, kg. \quad -(47)$$

This is compared with other estimates in Table 1:

| UFT Paper | m / kg |
|---|---|
| 150 | $3.35 \times 10^{-41}$ |
| 155 | $2.4 \times 10^{-38}$ |
| 244 | ~ electron mass |

| | |
|---|---|
| 245 | $\sim 10^{-37}$ |
| 264 | $1.04 \times 10^{-35}$ |
| 279 | $\sim 10^{-35}$ |
| 304 | $\sim 10^{-32}$ |

These estimates are much higher than the rest mass estimated form the de Broglie rest frequency:

$$\omega_0 = \frac{m_0 c^2}{\hbar} \quad -(48)$$

This must be less than the lowest observable frequencies of electromagnetic radiation, sub hertzian frequencies. For example electromagnetic radiation has been observed down to 3 Hz, equivalent to a photon rest mass of less than $10^{-51}$ kg. ∴

$$m_0 \sim \frac{\hbar}{c^2} \times (2\pi \times 3) \sim 10^{-51} \text{ kg}. \quad -(49)$$

Finally as in Note 310(8), and using:

$$\frac{\omega}{\omega_0} \doteq \exp\left(-\frac{\alpha \ell}{2}\right) \quad -(50)$$

from the Beer Lambert law it is found that photon mass itself obeys a Beer Lambert law:

$$m / m_0 = \gamma = \exp(-\alpha \ell / 2). \quad -(51)$$

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REFERENCES

{1} M. W. Evans, H. Eckardt, D. W. Lindstrom and S. J. Crothers, "Principles of ECE Theory" (open source as UFT281 to UFT288 on www.aias.us and to be published in softback with New Generation Publishing).

# Effect of photon mass on the Rayleigh Jeans density of states and the Planck distribution

M. W. Evans*, H. Eckardt†, G. J. Evans, T. Morris
Civil List, A.I.A.S. and UPITEC

(www.webarchive.org.uk, www.aias.us,
www.atomicprecision.com, www.upitec.org)

## 3 Graphical analysis and discussion

We did some calculations and graphs for the flux density as worked out in note 310(5) in detail. The flux density for the monochromatic radiation (Eq.(34) of section 2) has been plotted first. We chose three values of rest mass:

$$\begin{aligned} m_0 =& 10^{-35}\text{kg}, \\ & 10^{-40}\text{kg}, \\ & 10^{-50}\text{kg}, \end{aligned} \tag{52}$$

which correspons to frequencies of

$$\begin{aligned} \omega_0 =& \frac{m_0\, c^2}{\hbar} = 8.52 \cdot 10^{15}/\text{s}, \\ & 8.52 \cdot 10^{10}/\text{s}, \\ & 8.52/\text{s}. \end{aligned} \tag{53}$$

The graphs $\Phi(\omega)$ are linear on a double-logarithmic scale except near to the frequency $\omega_0$. In the vicinity of this frequency the flux drops to zero. Please notice that $\Phi$ differs of at least 20 orders of magnitude. For the smallest rest mass the dropping point is not on the plot. Obviously the flux density is defined above as well as below the de Broglie frequency $\omega_0$. Above this frequency it remains essentially zero because of the exponential factor exp(y) in the formula that dominates here for all de Broglie frequencies. This can be seen from Fig. 2 where the flux density is extremely small above $10^{16}$/s. Below this frequency, the photon mass effects a second cut-off in the flux density.

The behaviour is quite different for a poly-chromatic beam (Eq.(31)). We used the high-temperature approximation here, then we can approximate

$$\exp(y) \approx 1 + y \tag{54}$$

*email: emyrone@aol.com
†email: mail@horst-eckardt.de

and the integral (31) is simply solveable, giving

$$\Phi = c\frac{E}{V} = \frac{\left(\omega_1^2 - \omega_0^2\right)^{\frac{3}{2}} k\,T}{3\,\pi^2\,c^3}. \tag{55}$$

However the flux density is only defined here for $\omega > \omega_0$, otherwise the integrand is not defined. The graph (Fig. 3) is a kind of counterpart of the mono-chromatic case, see second figure. It raises linearly on a double-logarithmic scale, but the flux density is higher by orders of magnitude.

One can invert eq.(55) to obtain the photon mass in dependence of $\omega$ and $\Phi$. This gives an equation with six solutions. The only positive, real-valued solution is:

$$\omega_0 = \frac{\sqrt{\omega^2\,k^{\frac{2}{3}}\,T^{\frac{2}{3}} - 9^{\frac{1}{3}}\,\pi^{\frac{4}{3}}\,c^{\frac{4}{3}}\,\Phi^{\frac{2}{3}}}}{k^{\frac{1}{3}}\,T^{\frac{1}{3}}}. \tag{56}$$

This has been graphed in an $\omega/\Phi$ 3D plot in Fig. 4. As can be seen, the mass varies mainly with $\omega$, less with $\Phi$.

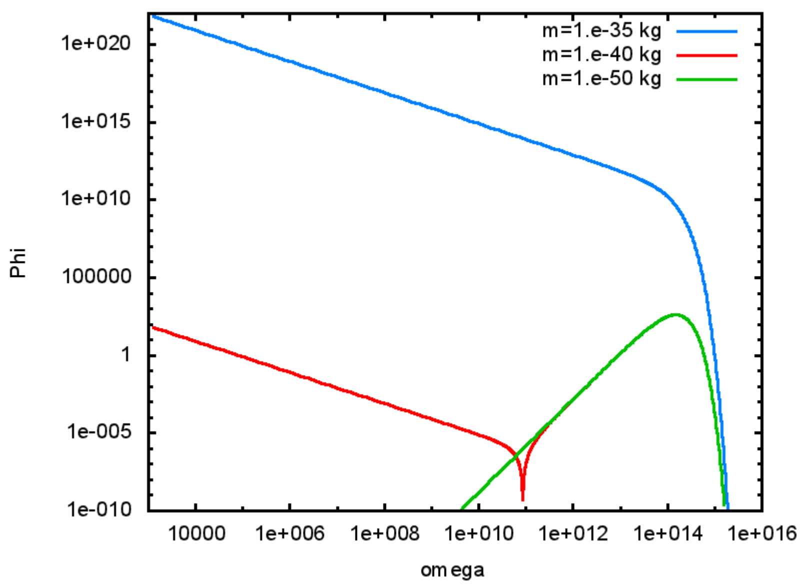


Figure 1: Flux density $\Phi(\omega)$ for monochromatic radiation, three rest mass values.

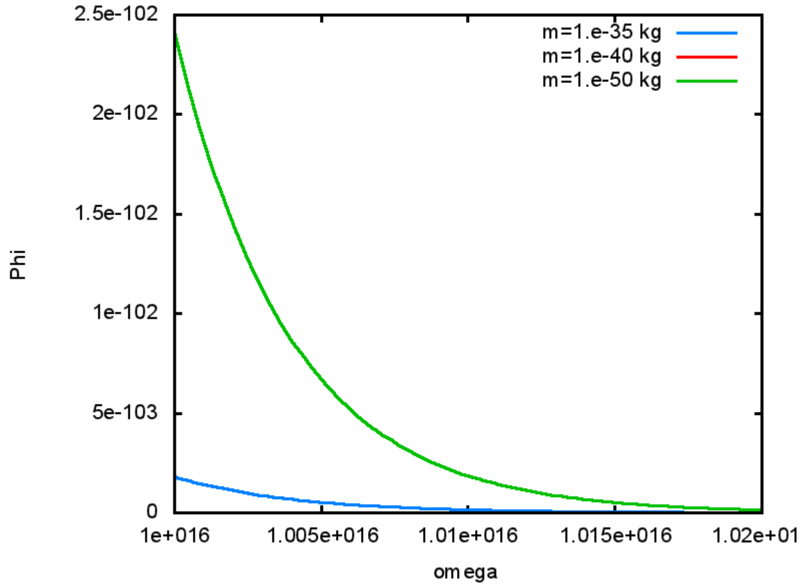


Figure 2: Flux density $\Phi(\omega)$ for monochromatic radiation, above temperature cut-off.

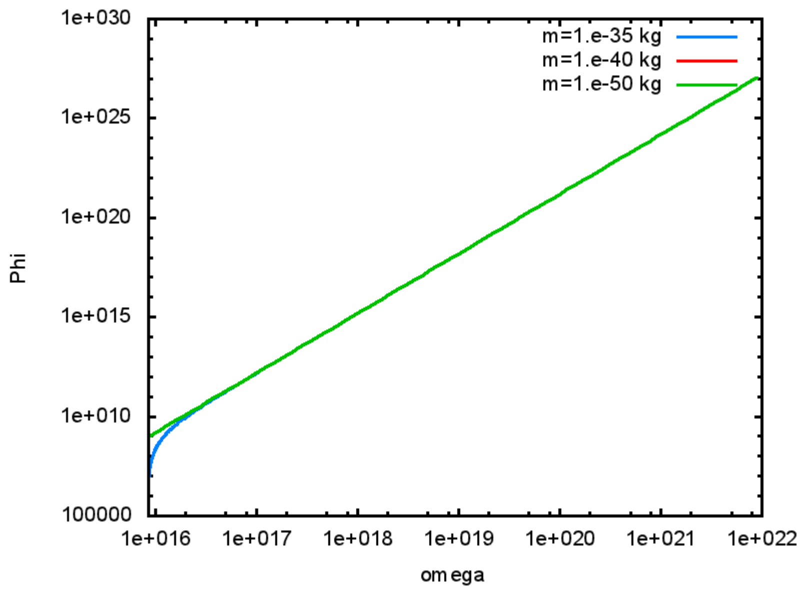


Figure 3: Flux density $\Phi(\omega)$ for poly-chromatic radiation, high-temperature limit, above rest mass cut-off.

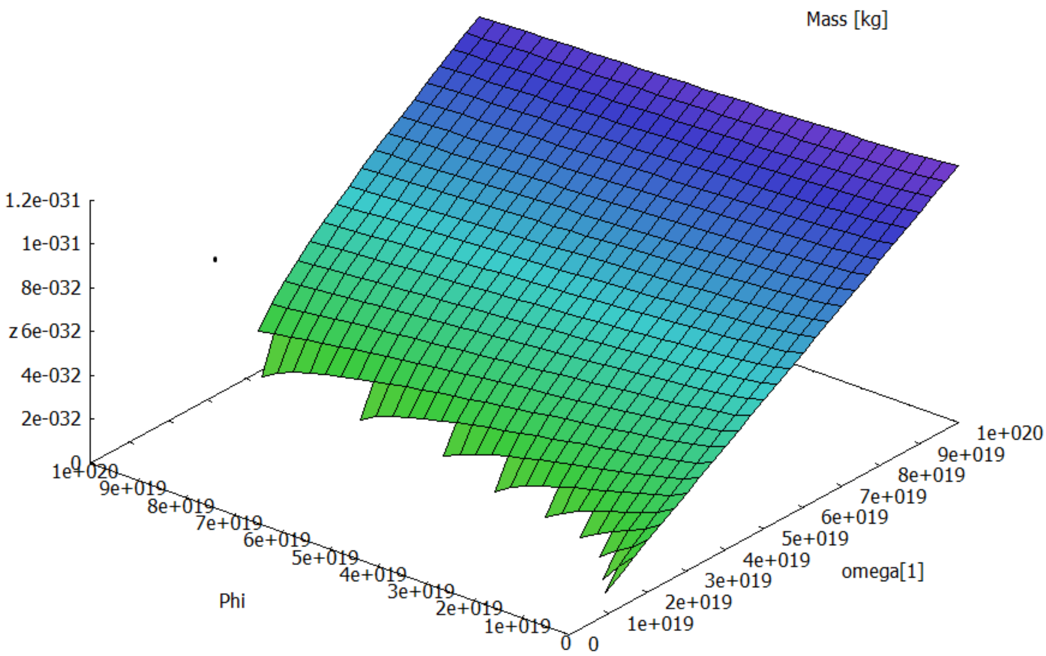


Figure 4: Rest mass from Eq.(56) in high-temperature limit.

{2} M .W. Evans., Ed. J. Found. Phys. Chem., (Cambridge International Science Publishing, CISP, www.cisp-publishing.com, and open source UFT papers on www.aias.us).

{3} M .W. Evans, Ed., "Definitive Refutations of the Einsteinian General Relativity" (CISP 2012 and open source on www.aias.us).

{4} M. W. Evans, S. J. Crothers, H. Eckardt and K. Pendergast, "Criticisms of the Einstein Field Equation" (CISP 2010 and open source as UFT301 on www.aias.us)

{5} M. W. Evans, H. Eckardt, and D. W. Lindstrom, "Generally Covariant Unified Field Theory" (Abramis Academic 2005 to 2012 and open source on the www.aias.us website).

{6} L. Felker, "The Evans Equations of Unified Field Theory" (Abramis 2007 and open source as UFT302 on www.aias.us).

{7} M .W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific, 2001).

{8} M. W. Evans and S. Kielich, Eds., "Modern Nonlinear Optics" (Wiley, New York, 1992, 1993, 1997 and 2001).

{9} M. W. Evans and J. - P. Vigier, "The Enigmatic Photon" (Kluwer, Dordrecht 1994 to 2002 and open source on www.aias.us).

{10} H. Eckardt, Ed., "ECE Engineering Model" (UFT303 on www.aias.us).

{11} M .W. Evans, "The Book of Scientometrics" (open source as UFT307 on www.aias.us).

{12} M. W. Evans and A. A. Hasanein, "The Photomagneton in Quantum Field Theory" (World Scientific, 1994).